Joey Leonard Justin Yaron

Thm 3.5

(1) $a \cdot 0_R = 0_R = 0_R \cdot a$, particularly $0_R \cdot 0_R = 0_R$

Proof: Since $0_R + 0_R = 0_R$

the distributive law shows:

$$a \cdot 0_R + a \cdot 0_R = a(0_R + 0_R)$$

$$a(0_R + 0_R) = a(0_R) = a(0_R) + 0_R$$

applying theorem 3.4 to the first and last parts of that equation shows that $a \cdot 0_R = 0_R$.

the proof that $0_R \cdot a = 0_R$ is similar

Emma Cobian
Austin Wilcox

Homework

(2) Prove $a(-b) = -ab$ and $(-a)b = -ab$

$a(-b) = -ab$

We notice that $-ab$ is the unique solution of

$ab + x = 0 \quad (x = -ab)$

Check if $a(-b)$ is a solution of $ab + x = 0$. If so then $-ab = a(-b)$ because $-ab$ is a unique solution.

If we plug $a(-b)$ in for x we get:

$ab + a(-b) = 0$

then by the distribution law

$a(b + (-b)) = 0$

$a(0) = 0$

We can use part 1 to then show that

$-ab = a(-b)$.

$(-a)b = -ab$

We notice that $-ab$ is the unique solution of

$ab + x = 0 \quad (x = -ab)$

Check if $(-a)b$ is a solution of $ab + x = 0$. If so then $-ab = (-a)b$ because $-ab$ is a unique solution.

If we plug $(-a)b$ in for x we get:

$ab + (-a)b = 0$

then by the distribution law

$\cancel{b}(a + (-a))b = 0$

$\cancel{b}(0)b = 0$

We can use part 1 to then show that

$-ab = (-a)b$.

3.5 #3

Katie Hollen
Logan Schmidt

given $-a + x = 0$

we know $a + -a = 0_R$

So $a + -a + x = a + 0_R$

So $x = a$

also

$-a + -(-a) = 0_R$

So $-(-a) + -a + x = -(-a) + 0_R$

So $x = -(-a)$ and $x = a$

By Theorem 3.3 There is only one unique solution To $-a + x = 0$

So $a = -(-a)$

Zac Weber & Amy Rice

Thm. 3.5 #4

(4) $-(a+b) = (-a)+(-b)$

By definition, $-(a+b)$ is the unique solution of $(a+b)+x = 0_R$, but $(-a)+(-b)$ is also because addition is commutative so that

$x = (-a)+(-b)$ | $(a+b) + [(-a)+(-b)] = a+(-a)+b+(-b)$

$a+(-a) = 0$

$b+(-b) = 0$

$= 0_R + 0_R = 0_R$

Therefore, $-(a+b) = (-a)+(-b)$ because of uniqueness ∎

Theorem 3.5 (5) $-(a-b) = -a+b$

$-(a-b) = -a+b \Rightarrow -(a+(-b)) = -a-b$

$\underbrace{-(a+(-b)) + (a+(-b))}_{0} = -a-b+(a+(-b))$ add $(a+(-b))$ to both sides

$0 = \underbrace{-a+a}_{0} + \underbrace{b+-b}_{0}$ $x+(-x)=0$

$0 = 0$

So $-(a-b) = -a+b$

Book's

$-(a-b)$

$= -(a+(-b))$

$= (-a)+(-(-b))$ by Thm 3.5 (4)

$= -a+b$ by Thm 3.5 (3)

So, $-(a-b) = -a+b$

Thm 3.5 (Part 6)

Proof

Given: $a, b \in \mathbb{R}$

Prove $(-a)(-b) = ab$

$= -(a(-b)) \longrightarrow$ (Part 2 - Right side)

$= -(-ab) \longrightarrow$ (Part 2 - Left side)

Thus $= ab \longrightarrow$ (Part 3)

so $(-a)(-b) = ab$ ✓

Key

Part 2

$a(-b) = -ab$ $\qquad$ $(-a)b = -ab$

Part 3

$-(-a) = a$

Bailey Pierskalla, Carly Boyle

Theorem 3.5 part 7
By part 2

show $(-1_R)a = -a$ 3.5 (2)

$$(-1_R)a = (-1_R a) = -(1_R a) = -(a) = -a$$

Because multiplication is associative
by part 2