show this

$\mathbb{Z}[i]$ subset of complex numbers $\mathbb{C}$

$$= \{a+bi \mid a, b \in \mathbb{Z}\}$$

is an integral domain:

Closure

Let $a+bi,\ c+di \in \mathbb{Z}[i]$

1. $(a+bi)+(c+di) =$
$(a+c)+(bi+di) =$
$(a+c)+(b+d)i \in \mathbb{Z}[i]$

6. $(a+bi)(c+di) =$
$ac+adi+bci+bdi^2$
$= ac+bd(-1)+adi+bci$
$= (ac-bd)+(ad+bc)i \in \mathbb{Z}[i]$

Automatically true because in $\mathbb{C}$

2, 3, 7, 8, 9

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4. $a+bi+\underbrace{0+0i} \overset{?}{=} a+bi$ ✓ $0_R = 0+0i$

5. $a+bi+\underbrace{-a+-bi} = 0+0i$ $-a+-bi \in \mathbb{Z}[i]$

10. $(a+bi)(1+0i) = a+bi$ $1+0i = 1_R$

11. Suppose $(a+bi)(c+di) = 0+0i$
$\Leftrightarrow ac+adi+bci+bdi^2 = 0+0i$
$\Leftrightarrow \underline{(ac-bd)}+\underline{(ad+bc)i} = \underline{0}+\underline{0i}$
$ac-bd=0$ $ad+bc=0$

Suppose $a \neq 0$ (show $c=d=0$)

Prove: if $a \neq 0$ then $c+di=0$

also need: if $b \neq 0$ then $c+di = 0$

Homework

$(ac - bd) + (ad + bc)i = 0 + 0i$

then $ac - bd = 0$ $\quad ad + bc = 0$

Suppose a is not 0 (show $c = d = 0$)

$ac - bd = 0$ $\quad bc = -ad$

$ac^2 - bcd = 0$

$ac^2 - (-ad)d = 0$

$a(c^2 + d^2) = 0$

since $a \neq 0$, then $c^2 + d^2 = 0$

$\Rightarrow c, d = 0$

~~Homework pg 56 # 24~~ (3.1)

22. ring R elements: $\mathbb{Z}$

$a \oplus b = a+b-1$ $\qquad$ $a \odot b = a+b-ab$

Closure is OK (1, 6 ✓)

2. $\oplus$ is associative

$a \oplus (b \oplus c) = a \oplus (b+c-1) = a+(b+c-1)-1 = a+b+c-2$

$(a \oplus b) \oplus c = (a+b-1) \oplus c = a+b-1+c-1 = a+b+c-2$

so $a \oplus (b \oplus c) = (a \oplus b) \oplus c$

do 3

check it

4. $a \oplus 0_R = a$ $\qquad$ $a \oplus 1 = a+1-1 = a$

$a + 0_R - 1 = a$ $\qquad$ $1 = 0_R$

$0_R = 1$

Homework: for # 22, check axioms

3

5 $\to$ $a \oplus x = 0_R$

$a \oplus x = 1$

7

8

9

10 $\to$ find 1_R