

Extra problem for the first isomorphism theorem:

Given that the function $\phi: \mathbb{Q}[x] \rightarrow \mathbb{Q}[\sqrt{3}]$ given by $\phi(f) = f(\sqrt{3})$ is well defined:

- a. Prove that ϕ is a homomorphism
- b. Prove that ϕ is a surjection (onto mapping)
- c. Describe $\ker(\phi)$ as a principal ideal and explain how you got that description.
- d. Tell what we can therefore conclude using the first isomorphism theorem (for rings).