R is a ring, I is an ideal

$R/I = \{ a+I \mid a \in R \}$

does it make sense to add?

$a+I,\ b+I \in R/I$

$(a+I)+(b+I) = (a+b)+I$ ← what we want

$(a+I)+(b+I) = \{ x+y \mid x \in a+I,\ y \in b+I \}$

part 1:

prove $(a+I)+(b+I) \subseteq a+b+I$

$n \in (a+I)+(b+I)$

so $n = x+y$ and $x \in a+I$
$y \in b+I$

so $\left.\begin{array}{l} x = a+i \\ y = b+j \end{array}\right\}$ where $i,j \in I$

so $n = a+i+b+j$
$= a+b+\underbrace{i+j}_{\in I} \in (a+b)+I$

$(a+I)+(b+I) \subseteq (a+b)+I$

part 2:

prove $a+b+I \subseteq (a+I)+(b+I)$

$n \in a+b+I$

$n = a+b+i$
$= \underbrace{a+0}_{\in a+I} + \underbrace{b+i}_{\in b+I} \in (a+I)+(b+I)$

Homework:

① show that multiplication makes sense

prove $(a+I)(b+I) = ab + I$

define: $(a+I)(b+I) = \{(a+i)(b+j) \mid i, j \in I\}$

② prove $0+I$ is the zero element:

to prove:
$(0+I)+(a+I) = (a+I)$

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③ Prove Theorem 6.10

6.2 (pg 159) #4