Abstract Algebra final exam topics:

1. Ideals: Show that a given subset of a ring is an ideal. For example, you might be asked to prove from the definition that $3\mathbb{Z} = \{3n \mid n \in \mathbb{Z}\}$ is an ideal. Proving it from the definition means that you can't just say: it's a principal ideal and therefore it's an ideal, instead you need to show it's a subring that absorbs ring elements under multiplication.

2. Quotient rings: Given a ring and an ideal, you might be asked to:

- list all of the cosets
- add or multiply two cosets
- prove that if two cosets share an element, then they are identical sets.
- In a quotient ring of $\mathbb{Q}[x]$, find an element of a coset with smallest degree.

3. First isomorphism theorem: Given a ring and a function, you might be asked to:

- Prove that the function is well defined
- Prove that the function is surjective (onto)
- Prove that the function is a homomorphism
- Find the kernel of the function
- State what you can conclude by using the first isomorphism theorem

Note: you are most likely to be asked about rings that are:

- A subset of $\mathbb{R}, \mathbb{C}$ or $\mathbb{Z}$ (including rings such as $\mathbb{Q}[\sqrt{2}]$ or $\mathbb{Q}[\sqrt{3}]$)
- A subset of $\mathbb{Z}_n$
- A subset of $\mathbb{Q}[x]$ or $\mathbb{Z}[x]$

4. Groups: Do computations in the groups S_n or D_n

5. Prove that a set with a given operation is or is not a group. In particular, you may be asked to prove that a cyclic subgroup is a group using the definition of a group.

6. Cyclic subgroups: List the elements in a finite cyclic subgroup.

In addition, you may be asked to solve some problems or prove some theorems from earlier in the semester, or possibly adapt (shorten) a proof so that it works for a groups. For example, theorems 7.5 and 7.6 are the group version of theorems we have proved for rings, and the process of proving them is identical to theorems we have already proved.

There are also likely to be a small number (perhaps 2) of problem from chapter 3.