Thm 1.5 Let p be an integer with $p \neq 0, \pm 1$. Then p is prime if and only if p has the property:

whenever $p \mid bc$ then $p \mid b$ or $p \mid c$

Step 1 Let p be a prime number

So I know: $p \neq 0, \pm 1$

And if $d \mid p$ then $d = \pm 1$ or $d = \pm p$

Let $b, c \in \mathbb{Z}$ such that $p \mid bc$

so $\boxed{bc = pr}$ for some $r \in \mathbb{Z}$

~~Suppose $p \nmid b$ p does not divide b~~ didn't need it

Let $(p, b) = e$

$e \mid p$ $\quad e \mid b$

$e = pv + bu$

either $e = \pm 1$ or $e = \pm p$

$\pm 1 = pv + bu$

$\pm c = cpv + \boxed{cb}u$

$\pm c = cpv + pru$

$\pm c = p(cv + ru)$

$p \mid c$

$b = en$

$b = \pm p \cdot n$

so $p \mid b$

Suppose P is not prime

Let d be a divisor of p, $p = dt$

then $p \mid dt$ $(p \cdot 1 = d \cdot t)$

Using the property:

Since $p \mid dt$, then $p \mid d$ OR $p \mid t$

Since $p = dt$, $-p \leq d \leq p$

because $p \mid d$ $-d \leq p \leq d$

then $d = \pm p$

making $t = \pm 1$ (because $p = dt$)

CONTRADICTION → Therefore P IS prime