If $a, b \neq 0$

and $a \mid b$ and $b \mid a$

then $b = \pm a$

Proof given $a, b \neq 0$

such that $a \mid b$ and $b \mid a$

then $|a| \,\big|\, |b|$ and $|b| \,\big|\, |a|$

So $|a| \leq |b|$ and $|b| \leq |a|$

So $|a| = |b|$

and $b = \pm a$ $\square$

Notice if $a, b > 0$

and $a \mid b$, $b \mid a$

then $a = b$

If Sets A, B

$A \subseteq B$ and $B \subseteq A$

then $A = B$

If $a \mid (b+c)$ and $(b,c)=1$

To prove: $(a,b)=1$

Proof: Given $a \mid (b+c)$ and $(b,c)=1$

$1 = bu + cv$

So, $b+c = a \cdot k$ (for some $k \in \mathbb{Z}$)

$c = ak - b$

Let $(a,b) = e$

$a = eN \quad b = eM \quad e = aU + bV$

$1 = bu + cv$

$1 = eMu + cv$

$1 = eMu + (ak-b)v$

$1 = eMu + (eNk - eM)v$

$1 = e(Mu + (Nk-M)v)$

$1 = e(\underbrace{\quad\quad\quad}\,)$

$e \mid 1$

$e = 1$

$(a,b) \mid 1$

$(a,b) = 1$

We started #18 in class this way

18. If $c>0$

Prove $(ca, cb) = c(a,b)$

Proof: Let $c>0$ $\quad a,b,c \in \mathbb{Z}$

Let $(ca, cb) = d$. and let $(a,b) = e$

So $ca = dn$ ☆ $\quad$ + $a = eN$

$cb = dm$ ☆ $\quad$ + $b = eM$

$e = aU + bV$ ☆

+ $d = cau + cbv$

☆ use these to get

$ce = d(\qquad)$

use these to get

$d = ce(\qquad)$

$ce = d(\quad)$ $\quad d = ce(\quad)$

$d \mid ce$ and $ce \mid d$.

$d = ce$. $\therefore$

$(ca, cb) = c(a,b)$