Theorem 1.5 step 2: Given an integer $p$ such that :

- $p \neq 0, \pm 1$
- whenever $p \mid bc$ then $p \mid b$ or $p \mid c$

Prove that $p$ is prime.

*proof:*

$p$ already has the property $p \neq 0, \pm 1$

Let $d \mid p$

which means $p = dk$ for some $k \in \mathbb{Z}$

so $p \mid dk$ and $k \mid p$

then by the second property, $p \mid d$ or $p \mid k$

Case 1: $p \mid d$

then $p \mid d$ and $d \mid p$ so $d = \pm p$

Case 2: $p \mid k$

then $p \mid k$ and $k \mid p$ so $k = \pm p$

So then $p = \pm p \cdot d$ and hence $d = \pm 1$

So any divisor of $p$ is either $\pm p$ or $\pm 1$

And by definition $p$ is prime.