Some things about greatest common divisors that you should know and know how to prove:

Theorem GCD 1 If $a, b \neq 0$ then $1 \leq (a,b) \leq |b|$

proof: 1 divides evenly into every integer, so $1 | a$ and $1 | b$ and 1 is a common divisor of a and b, hence the greatest common divisor (a,b) must be at least at large as 1: $1 \leq (a,b)$

The greatest common divisor (a,b) must be a divisor of b, so $(a,b) \big| |b|$ and hence $(a,b) \leq |b|$. Thus $1 \leq (a,b) \leq |b|$

Theorem GCD 2 If $a \neq 0$ and p is prime, then $(a,p) = 1 \text{ or } p$

proof: The greatest common divisor (a,p) must be a divisor of p, so (by definition of prime) $(a,p) = \pm 1, \pm p$ and 1 divides evenly into every integer so $1 \leq (a,p)$. Thus, $(a,p) = 1 \text{ or } p$

Theorem GCD 3 If $a \neq 0$, p is prime and $p \nmid a$ then $(a,p) = 1$

proof: The greatest common divisor (a,p) must be a divisor of p, so (by definition of prime) $(a,p) = \pm 1, \pm p$ and 1 divides evenly into every integer so $1 \leq (a,p)$. Thus, $(a,p) = 1 \text{ or } p$.

Also, (a,p) must be a divisor of a, so $(a,p) | a$. We are given $p \nmid a$, so $(a,p) = 1$.