

Some theorems you need to know how to prove:

Theorem M1. If  $p$  is prime, then every non-zero element in  $\mathbb{Z}_p$  is a unit

Theorem M2. If  $p$  is prime, then  $\mathbb{Z}_p$  has no zero-divisors.

Theorem M3. If  $p$  is prime, and  $a$  and  $b$  are constants and  $a \neq 0$  in  $\mathbb{Z}_p$ , then  $ax + b = 0$  has a solution in  $\mathbb{Z}_p$ .

Theorem M4. If  $p$  is prime, and  $a, b$  and  $c$  are constants and  $a \neq 0$  in  $\mathbb{Z}_p$ , then  $ab = ac$  implies  $b = c$ .

Theorem M5. If  $n$  is not prime, then there exists a zero-divisor in  $\mathbb{Z}_n$ .

Theorem M6. If  $a < n$  and  $(a, n) > 1$  then  $a$  is a zero-divisor in  $\mathbb{Z}_n$ .

Theorem M7. If  $a < n$  and  $(a, n) = 1$  then  $a$  is a unit in  $\mathbb{Z}_n$ .

Examples you need to know:

1. Find numbers  $a, b, c, n$  such that  $ab = ac$ , and  $a \neq 0$  but  $b \neq c$  in  $\mathbb{Z}_n$
2. Find numbers  $a, b, n$ , where  $a \neq 0$ , such that  $ax + b = 0$  has more than one solution in  $\mathbb{Z}_n$ .
2. Find numbers  $a, b, n$ , where  $a \neq 0$ , such that  $ax + b = 0$  has no solutions in  $\mathbb{Z}_n$ .