Practice problems:

1. For $f:\mathbb{Z}_{20}\to\mathbb{Z}_5$ such that $f([n]_{20})=[n]_5$

a. Prove f is a well-defined function

b. Prove that f is a homeomorphism

c. Find and describe the kernel $\ker(f)$

d. Find and describe the elements of $\mathbb{Z}_{20}/\ker(f)$

e. Prove that f maps $\mathbb{Z}_{20}$ **onto** $\mathbb{Z}_5$

f. Given the previously proven properties of f, what can we conclude using the first isomorphism theorem (Thm 6.13)?

g. Define the natural isomorphism corresponding to the result in part (f)

2. Let $R=\left\{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \middle| a,b,c\in\mathbb{Q}\right\}$, and let $f:R\to\mathbb{Q}$ such that $f\left(\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right)=a$

a. ~~Prove f is a well-defined function~~ because elements in the domain have only one form, it's clear that the function is well defined, so I won't ask you to prove it.

b. Prove that f is a homeomorphism

c. Find and describe the kernel $\ker(f)$

d. Find and describe the elements of $R/\ker(f)$

e. Prove that f maps R **onto** $\mathbb{Q}$

f. Given the previously proven properties of f, what can we conclude using the first isomorphism theorem (Thm 6.13)?

g. Define the natural isomorphism corresponding to the result in part (f)