351 examples

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Given $a \mid b+c$ and $(b,c)=10$
prove $(a,b) \mid 10$

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proof:

given $a \mid b+c \longrightarrow b+c=an$ for some $n \in \mathbb{Z}$

$(b,c)=10 \longrightarrow 10 \mid b \quad 10 \mid c \quad bu+cv=10$

$b=10s \quad c=10t$ for some $s,t,$
$u,v \in \mathbb{Z}$

also name [Let $d=(a,b)$

$d \mid a \quad d \mid b \quad aj+bk=d$

$a=dp \quad b=dq$ for some $p,q,j,k \in \mathbb{Z}$

$bu+cv=10 \longleftarrow$ equation $=10$

$b=dq \rightarrow dqu+cv=10$ need something with c & d

$b+c=an \rightarrow c=an-b$

$c=dpn-dq$

$dqu+dpn-dq=10$

$d(\underbrace{qu+pn-q}_{\in \mathbb{Z}})=10$

so $\rightarrow 10=d(\quad)$

goal [$d \mid 10$ QED

Given $(a,b)=(a,5a+b)$

proof:

name things
$$\left[\begin{array}{l} \text{Let } c=(a,b) \\ \text{Let } d=(a,5a+b) \end{array}\right.$$

write equations
$$\left[\begin{array}{l} C=(a,b) \text{ so} \\ c|a \quad c|b \quad c=au+bv \\ a=cj \quad b=ck \end{array}\right.$$

for some $j,k,u,v,p,q,n,m \in \mathbb{Z}$

$$\left[\begin{array}{l} d=(a,5a+b) \\ d|a \quad d|5a+b \quad d=am+(5a+b)n \\ a=dp \quad 5a+b=dq \end{array}\right.$$

use $c=au+bv$ with $a=dp$, $5a+b=dq$

$b=dq-5a$

$b=dq-5dp$

$c=dpu+(dq-5dp)v$

$c=d(pu+qv-5pv)$

use $d=am+(5a+b)n$ with $a=cj$, $b=ck$

$d=cjm+(5cj+ck)n$

$d=c(jm+5jn+kn)$

goal
$$\left[\begin{array}{ll} d=c(\quad) & c=d(\quad) \\ c|d, & d|c \\ d=1 & \end{array}\right.$$

goal $\begin{cases} c \mid a \\ c = d \end{cases}$

QED

Example:

if $3u+5v=1$ then $(u,v)=1$

Proof:

given $3u+5v=1$

name [Let $d=(u,v)$ for some $j,k,s,t\in\mathbb{Z}$

equations [then $d|u$ $d|v$

$u=dj$ $v=dk$ $d=us+vt$

use $3u+5v=1$ with $u=dj$ $v=dk$

$3\cdot dj+5\cdot dk=1$

$d(3j+5k)=1$

Goal $\left\{ \begin{array}{l} 1=d(\quad\quad) \\ d \mid 1 \\ d=1 \end{array} \right.$

QED