

Some theorems you need to know how to prove:

Theorem M1. If p is prime, then every non-zero element in \mathbb{Z}_p is a unit

Theorem M2. If p is prime, then \mathbb{Z}_p has no zero-divisors. (B)

Theorem M3. If p is prime, and a and b are constants and $a \neq 0$ in \mathbb{Z}_p , then $ax + b = 0$ has a solution in \mathbb{Z}_p .

Theorem M4. If p is prime, and a, b and c are constants and $a \neq 0$ in \mathbb{Z}_p , then $ab = ac$ implies $b = c$.

Theorem M5. If n is not prime, then there exists a zero-divisor in \mathbb{Z}_n .

Theorem M6. If $a < n$ and $(a, n) > 1$ then a is a zero-divisor in \mathbb{Z}_n .

Theorem M7. If $a < n$ and $(a, n) = 1$ then a is a unit in \mathbb{Z}_n .

Proved (F):
if $1 < a < n$ and
 $a|n$
then a is a zero-div. in \mathbb{Z}_n

Examples you need to know:

1. Find numbers a, b, c, n such that $ab = ac$, and $a \neq 0$ but $b \neq c$ in \mathbb{Z}_n
2. Find numbers a, b, n , where $a \neq 0$, such that $ax + b = 0$ has more than one solution in \mathbb{Z}_n .
2. Find numbers a, b, n , where $a \neq 0$, such that $ax + b = 0$ has no solutions in \mathbb{Z}_n .

M8: In \mathbb{Z}_n if $a \neq 0$, then a is a unit or a 0-divisor, but not both.

M9(?): If $n > 2$ There are an even number of units in \mathbb{Z}_n

M10: 1 is a unit in \mathbb{Z}_n

Useful