**Ring property assignment**

1. Prove the following two theorems (these are exactly the same proofs we did for groups, so this is a review problem)

a. **Theorem 39:** The additive identity of a ring *R* is unique.

b. **Theorem 41:** For any element *a* of a ring *R*, the additive inverse of *a* is unique.

2. Fill in the missing steps to prove

**Theorem 38:** If  is a ring, and  then 

Proof:

Let  be a ring, let  and let 0 be the additive identity of *R*.

We know 

So (multiply both sides of the equation by *a* on the left)

So (use the distributive law)

The element  has an additive inverse: 

So (add the additive inverse to both sides of the equation)

So (simplify by adding )

Therefore 

Similarly, 

3. Finish the proof of:

**Theorem 45:** If then  and 

*proof*:

Let  be a ring, and let 

Because *R*, is closed, 

And because every element of *R* has an additive inverse in *R*, we know 

Because *R*, is closed, 

Consider 

so 

and  by the distributive property, and Theorem 38

so 

and 

therefore .

[next prove that ) in the same way]

4. Prove theorem 40. Be careful: the only element you know has a multiplicative inverse is the identity itself, so you will need to use the hint: *if there were two identities, what would their product be?*

**Theorem 40:** The multiplicative identity of a ring with identity (*R*) is unique. (Hint: if there were two identities, what would their product be?)

5. Using properties of rings, and Theorem 45, and given is a ring, and , prove that is a subring of *R*.