Ring property assignment

1. Prove the following two theorems (these are exactly the same proofs we did for groups, so this is a review problem)
a. **Theorem 39:** The additive identity of a ring R is unique.
b. **Theorem 41:** For any element a of a ring R, the additive inverse of a is unique.

2. Fill in the missing steps to prove
Theorem 38: If $R,+,\cdot$ is a ring, and $a \in R$ then $a \cdot 0 = 0 \cdot a = 0$
Proof:
Let $R,+,\cdot$ be a ring, let $a \in R$ and let 0 be the additive identity of R.

We know $0+0=0$

So (multiply both sides of the equation by a on the left)

So (use the distributive law)

Every element $a \cdot 0$ has an additive inverse $-(a \cdot 0) \in R$

So (add the additive inverse to both sides of the equation)

So (simplify by adding $-(a \cdot 0) + a \cdot 0 = 0$)

Therefore $a \cdot 0 = 0$

Similarly, $0 \cdot a = 0$

3. Prove theorem 40. Be careful: the only element you know has a multiplicative inverse is the identity itself, so you will need to use the hint: if there were two identities, what would their product be?

Theorem 40: The multiplicative identity of a ring with identity (R) is unique. (Hint: if there were two identities, what would their product be?)