Principal ideals, cosets and congruence in a polynomial ring

$x^2 - 2 \in \mathbb{Q}[x]$

$I = (x^2 - 2) = \{(x^2 - 2)p(x) \mid p(x) \in \mathbb{Q}[x]\}$
is a principal ideal in $\mathbb{Q}[x]$

$cR = \{cr \mid r \in R\}$ is a **principal ideal** of R

Congruence classes are cosets of I:

$[2x+1] = 2x+1+I = \{2x+1+(x^2-2)p(x) \mid p(x) \in \mathbb{Q}[x]\}$

$a + I = \{a + i \mid i \in I\}$ is a **coset** (also called equivalence class)

Polynomials are congruent if:

$f(x), g(x) \in \mathbb{Q}[x]$

$g(x) - f(x) \in I = (x^2 - 2)$

$g(x) - f(x) = (x^2 - 2)p(x)$

$x^2 - 2 \mid g(x) - f(x)$

a* is congruent to *b* modulo *I if $b + (-a) \in I$. We write $a \equiv b \pmod{I}$

The equivalence classes form a ring:

$$\mathbb{Q}[x]/(x^2-2) = \{f(x) + I \mid f(x) \in \mathbb{Q}[x]\}$$
$$= \{[f(x)] \mid f(x) \in \mathbb{Q}[x]\}$$

The quotient ring of *R* mod *I* is $R/I = \{a + I \mid a \in R\}$