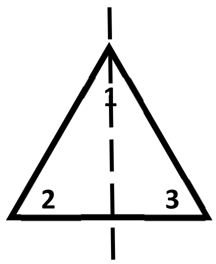
Function composition is associative

Example 1:	$f\circ(g\circ h)$ vs $(f\circ g)\circ h$
$f:\mathbb{R}\to\mathbb{R}$ s.t. $f(x)=x+2$ $g:\mathbb{R}\to\mathbb{R}$ s.t. $g(x)=x^3$ $h:\mathbb{R}\to\mathbb{R}$ s.t. $h(x)=5x$	$f\circ(g\circ h)=f(g\circ h(x))$ $g\circ h=g(h(x))=(5x)^3$ $f\circ(g\circ h)=f((5x)^3)=(5x)^3+2$ $(f\circ g)\circ h=f\circ g(h(x))$ $f\circ g=f(g(x))=x^3+2$ $f\circ g(h)=(5x)^3+2$

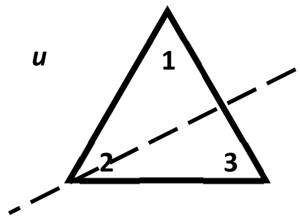
Example 2:

v



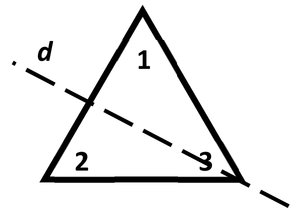
$1\to1$
$2\to3$
$3\to2$

u



$1\to3$
$2\to2$
$3\to1$

d



$1\to2$
$2\to1$
$3\to3$

$(v\circ u)\circ d=p\circ d=u$

$v\circ(u\circ d)=v\circ p=u$

$v\circ u$	$(v\circ u)\circ d$
$u \quad v$	$d \quad u \quad v$
$1\to3\to2$	$1\to2\to2\to3$
$2\to2\to3$	$2\to1\to3\to2$
$3\to1\to1$	$3\to3\to1\to1$

Abstract proof

Given functions $f:C\to D$ $\quad g:B\to C$ $\quad h:A\to B$

Note:

$f\circ(g\circ h)$

$$\overset{h}{A\to}\overset{g}{B\to}\overset{f}{C\to}D$$

$f\circ(g\circ h)(x)=f((g\circ h)(x))=f(g(h(x)))$

$(f\circ g)\circ h(x)=(f\circ g)(h(x))=f(g(h(x)))$

So $f\circ(g\circ h)=(f\circ g)\circ h$, and therefore, function composition is associative.