1. Given that you know (from an earlier assignment) that $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$

a. Prove that $f : \mathbb{Q}(\sqrt{2}) \to \mathbb{Q}(\sqrt{2})$ such that $f(a + b\sqrt{2}) = a - b\sqrt{2}$ is a homomorphism.

b. Prove that $g : \mathbb{Q}(\sqrt{2}) \to \mathbb{Q}(\sqrt{2})$ such that $g(a + b\sqrt{2}) = a + 2b\sqrt{2}$ is a not a homomorphism.