Abstract Algebra Notes 1/29/2020

Things you should now know about the Dihedral group:

- D_n (the dihedral group of degree *n*) is the group of symmetries of a regular *n*-gon, with operation function composition.
- This elements of this group are functions that map a regular *n*-gon to itself.
- The group operation is function composition.
- D_n consists of *n* rotations (one of which is the identity rotation of 0° or 360°) and *n* reflections
- The **order** of D_n (the number of elements in it) is 2*n*.
- Every reflection is its own inverse.
- The smallest rotation in a dihedral group is usually given the name *r* or ρ, and it is the rotation by $\dfrac{360°}{n}$
- $r^k = \underbrace{r \circ r \circ \ldots \circ r}_{k \text{ times}}$
- $r^n = e$ is the identity element of the group, and it is the function that sends every point of the *n*-gon to itself.
- The inverse of the rotation r^k is the rotation r^{n-k}
- D_n is not commutative

There are things that are fun to know about dihedral groups, that you now know enough to discover:

1. The order of an element a is the smallest natural number *k* such that $a^k = e$. What is the order of *r* in D_n?
2. Draw your regular *n*-gon so that it has a vertical line of symmetry, and call the reflection in that symmetry line *v*. Now, it's going to be true that $r \circ v \neq v \circ r$, but its also true that $r \circ v = v \circ r^k$ for some *k*. What is *k*?
3. Every reflection in D_n can be represented as $r^k \circ v$ for some *k*. Why is this always true?
4. Given $1 \leq k \leq n$, for every such *k*, there is another number *j* such that $r^k \circ v = v \circ r^j$. How are *j* and *k* related?

The Associative law: For an operation *, then * is **associative** if $a*(b*c) = (a*b)*c$ for all elements *a, b, c*.

5. Choose either addition or multiplication (of natural numbers) and explain why it is associative.
6. Write out three 2×2 matrices with all variables. Show by doing all of the long ugly calculations that matrix multiplication is associative. If you don't remember how to multiply matrices: Google it. This is messy enough that you're allowed to do this one with a partner: one of you do one side, the other do the other side, then compare.
7. Explain, using function notation, how we know that function composition in D_6 is associative (hint: this is done exactly the same way for showing that function composition in any context is commutative).
8. For each of these weird operations, decide if it is associative or not. Be prepared to explain how you decided:
    a) For the set of natural numbers, if *n* and *m* are natural numbers, then *n**m* is the least common multiple of *m* and *n*, so, for example 6*4=12. Is * associative?
    b) For the set of rational numbers, if *n* and *m* are natural numbers, then *n#m* is the number half-way in between (the average of the two), for example 4#6=5 and 2#3=2.5. Is # associative?
    c) For the set consisting of the integers, is the operation of subtraction associative?
    d) For the set of rational numbers, with operation $x*y = xy/2$, for example $3*4 = 6$ and $3*3 = 4.5$. Is * associative?