The two practice problems I asked you to work out at the end of class (due Friday, but not collected) are:

1. For $f : \mathbb{Z}_5 \to \mathbb{Z}_8$ such that $f([n]_5) = [n]_8$, prove that f is not a function

2. let $g : \mathbb{Z}_5 \to \mathbb{Z}_{10}$ such that $g([n]_5) = [2n]_{10}$, prove that g is a function.

Big Idea

The important understanding to make sense of both of these problems is that $[n]_5 \in \mathbb{Z}_5$ can be written in several ways, so $[2]_5 = [7]_5 = [12]_5 = [17]_5$ are all different ways of writing the same element of $\mathbb{Z}_5$. That's because an element of $\mathbb{Z}_5$ is a set (equivalence class) of integers.

How to start #1

In order for f or g to be a function, the rule has to do the same thing to the equivalence class set of $[n]_5$, no matter how you write it. Because $[2]_5 = [7]_5 = [12]_5 = [17]_5$, f is only a function if $f([2]_5) = f\left([7]_5\right) = f\left([12]_5\right) = f\left([17]_5\right)$. If you use the rule $f([n]_5) = [n]_8$ on the different representations of $[2]_5 = [7]_5 = [12]_5 = [17]_5$, you will find that you get elements in $\mathbb{Z}_8$ that don't just look different, they are actually in different equivalence classes in $\mathbb{Z}_8$. When you're proving something is not a function, you just need one example showing that it doesn't work, so by using the rule for f on two different versions of $[2]_5$, you will probably be able to show it is not a function.

How to start #2.

To prove something is a function, you can't just find one example that it works, you need to use algebra to show that every example works. That means you need a way to write down a generic element of $\mathbb{Z}_5$: I'm going to choose $[n]_5 \in \mathbb{Z}_5$, and then you need a way to write down all of the different representations of that equivalence class: $[n]_5 = \{n + 5k \mid k \in \mathbb{Z}\}$. I'm going to pick these two representations: $[n+5i]_5$ and $[n+5j]_5$ to be my variable-version names for two ways of writing the equivalence class.

To show that g is a function, you need to show that when you apply the rule $g([n]_5) = [2n]_{10}$ to both of these representations, you get the same element of $\mathbb{Z}_{10}$, which is the same as saying that their representations are equivalent mod 10. You can start by writing out what happens when you plug the two representations of $[n]_5$ in to g, and then show that they are in the same equivalence class mod 10, or you can start with the definition of equivalence mod 10.

How to finish #1:

This one, as I said above, only needs a single example:

$f([2]_5) = [2]_8$ but $f([7]_5) = [7]_8$

So $[2]_5 = [7]_8$

But $f([2]_5) \neq f([7]_8)$

This shows f is not a function.

How to finish #2:

Let $n+5i,\ n+5j \in [n]_5$

Then

$g([n+5i]_5) = [2(n+5i)]_{10} = [2n+10i]_{10}$

And

$g([n+5j]_5) = [2(n+5j)]_{10} = [2n+10j]_{10}$

Now, $[2n+10i]_{10} = [2n]_{10} + [10i]_{10} = [2n]_{10}$ And $[2n+10j]_{10} = [2n]_{10} + [10j]_{10} = [2n]_{10}$ So $g([n+5i]_5) = g([n+5j]_5)$ Thus g is a function.	Writing this another way: $(2n+10i)-(2n+10j) = 10i-10j = 10(i-j)$ So $2n+10i \equiv 2n+10j \pmod{n}$ So $[2n+10i]_{10} = [2n+10j]_{10}$ So $g([n+5i]_5) = g([n+5j]_5)$ Thus g is a function.