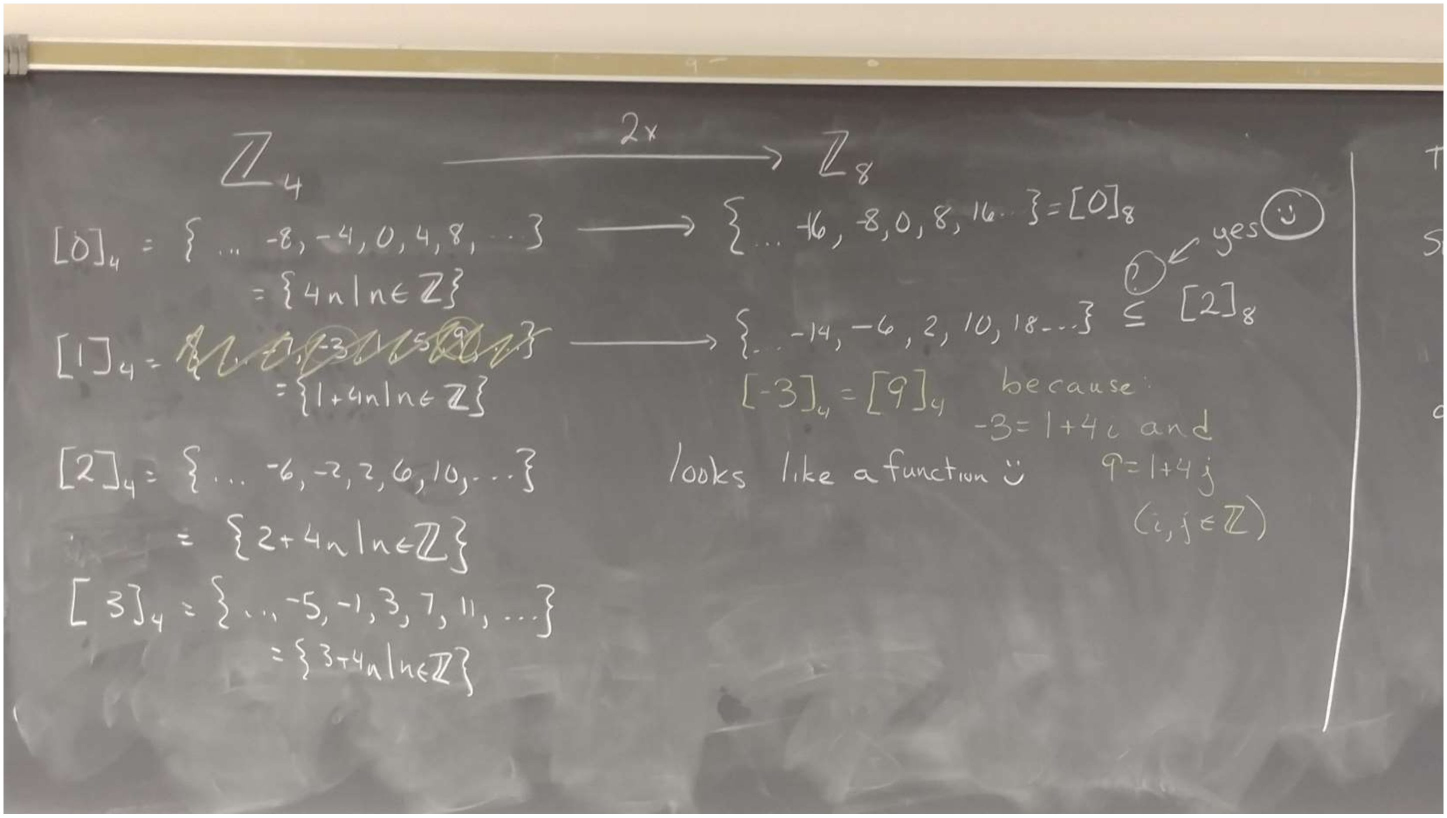
Math 351 Notes (with additional extension) 3-6-2020

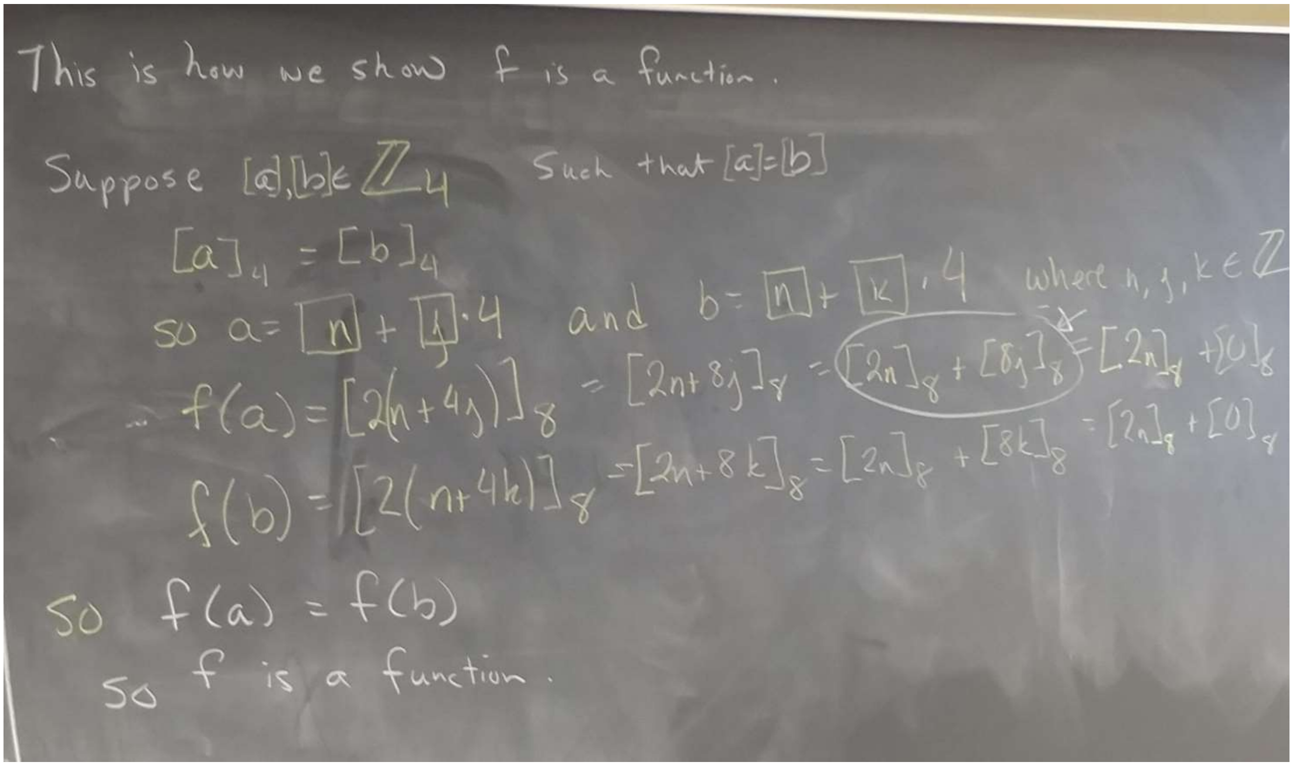
Mod numbers are actually sets of integers, In order to be a function on mod numbers, every number in the equivalence class set of a mod number has to be mapped to the same place.



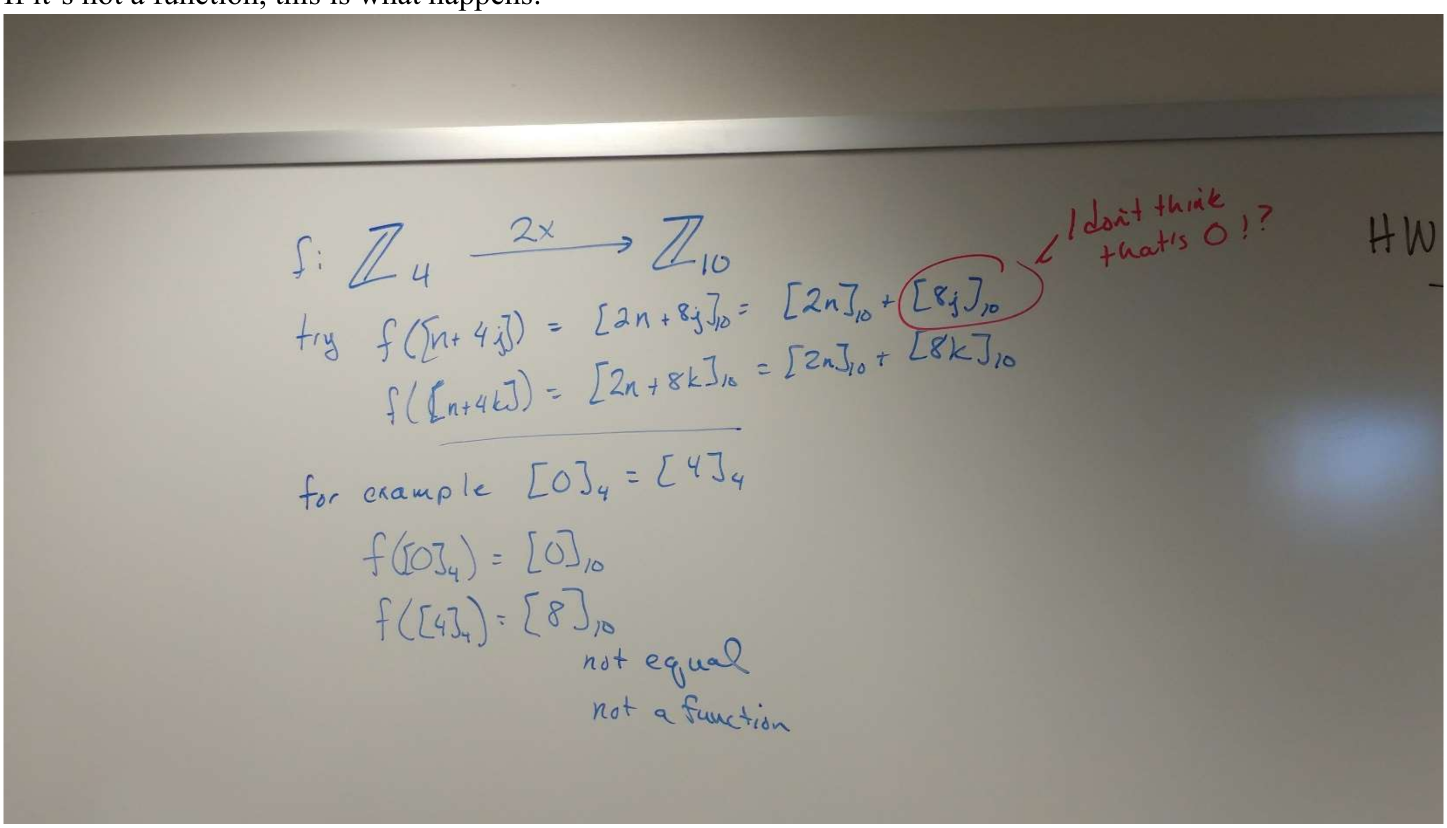

The strategy for showing a function is 1-to-1 is very similar to the strategy for showing that a potential function is a function:

| | |
|---|---|
| To show 1-to-1<br>suppose $a, b \in$ domain<br>s.t $f(a) = f(b)$<br>do work<br>$\downarrow$<br>so $a = b$<br>so $f$ is 1-to-1 | to show it's a function:<br>Suppose $a, b \in$ domain<br>s.t. $a = b$<br>do work<br>$\downarrow$<br>$f(a) = f(b)$<br>so $f$ is a function |

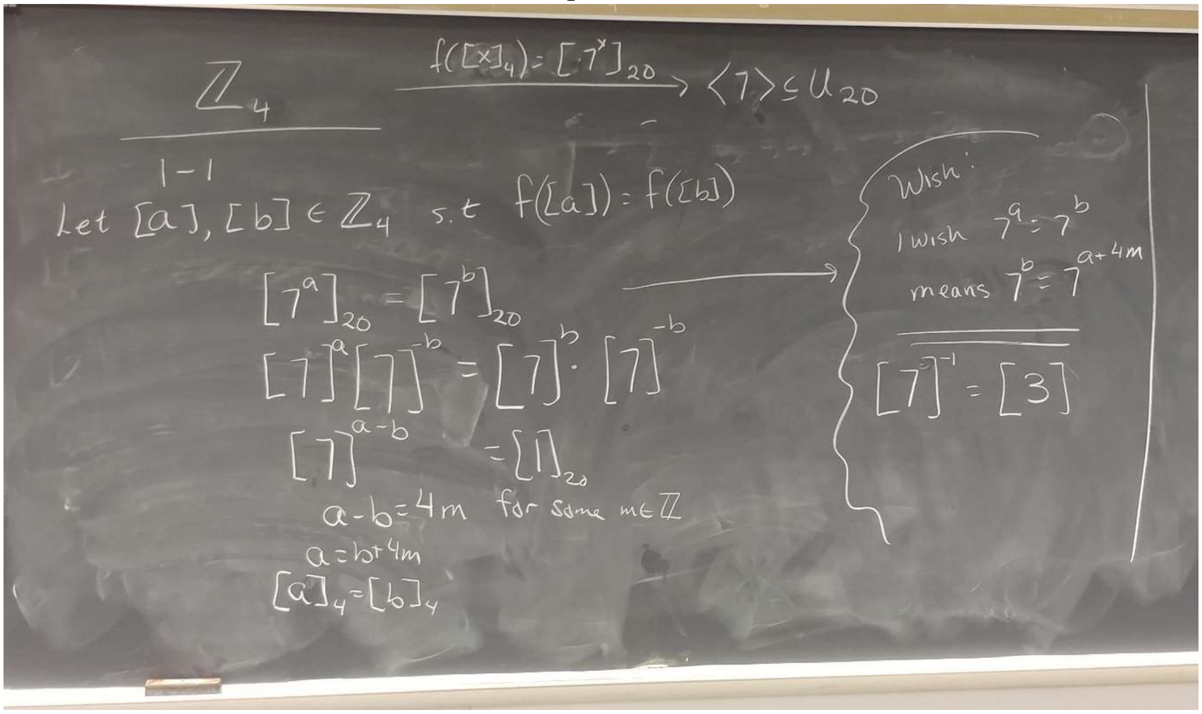
Here we used that strategy to prove that our first example potential function is really a function.




If it’s not a function, this is what happens:

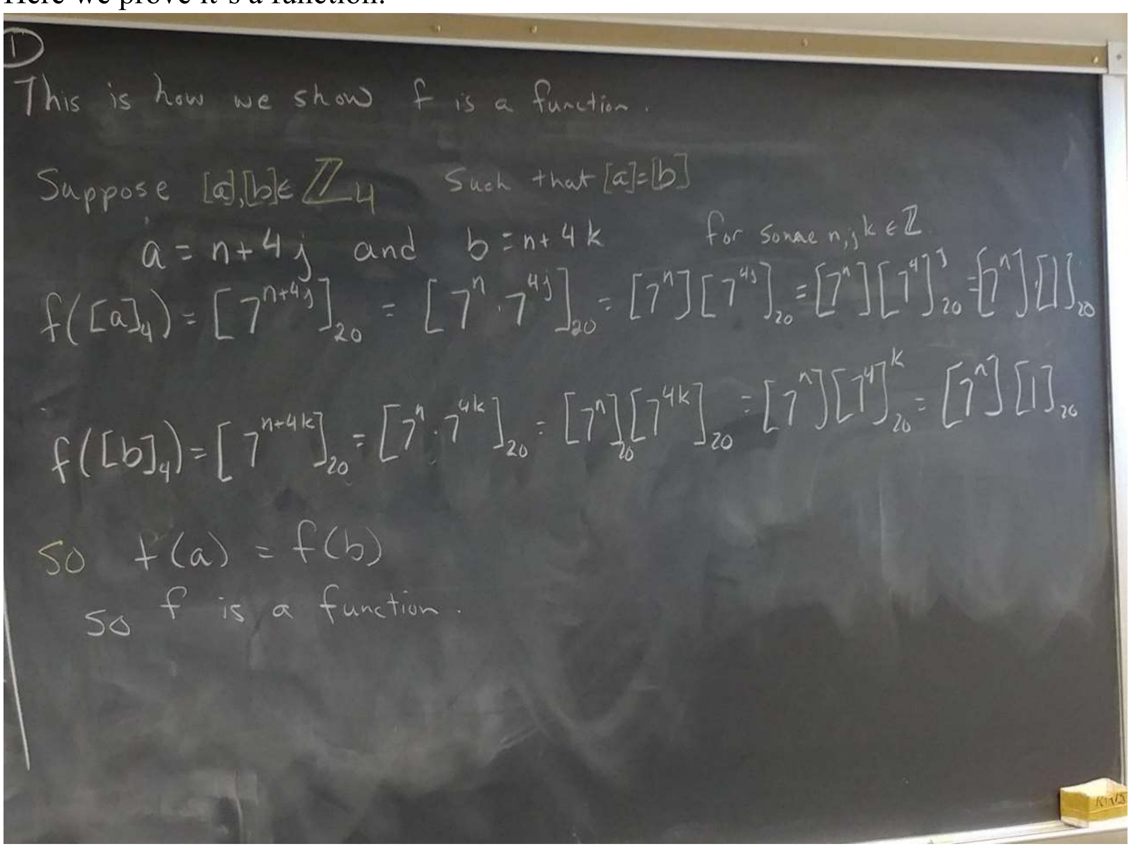


Here’s most of a proof that something is an isomorphism:
This is the definition of the relation we will prove is a function:

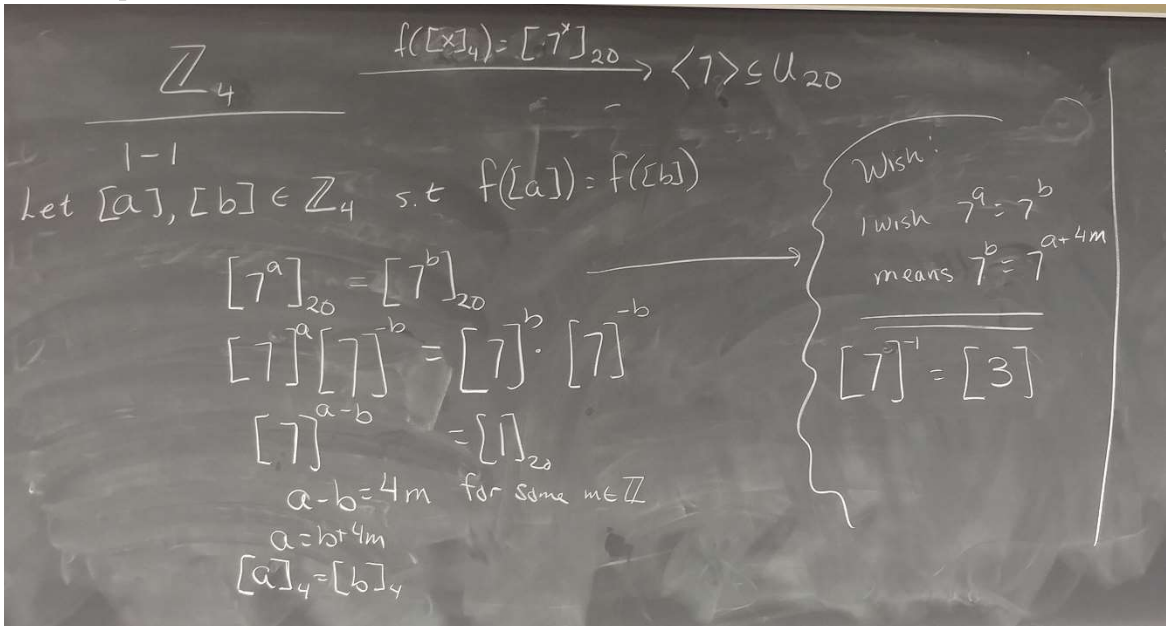



Recall from earlier that $7^2 \equiv 9 \pmod{20}$, $7^3 \equiv 3 \pmod{20}$, $7^4 \equiv 1 \pmod{20}$

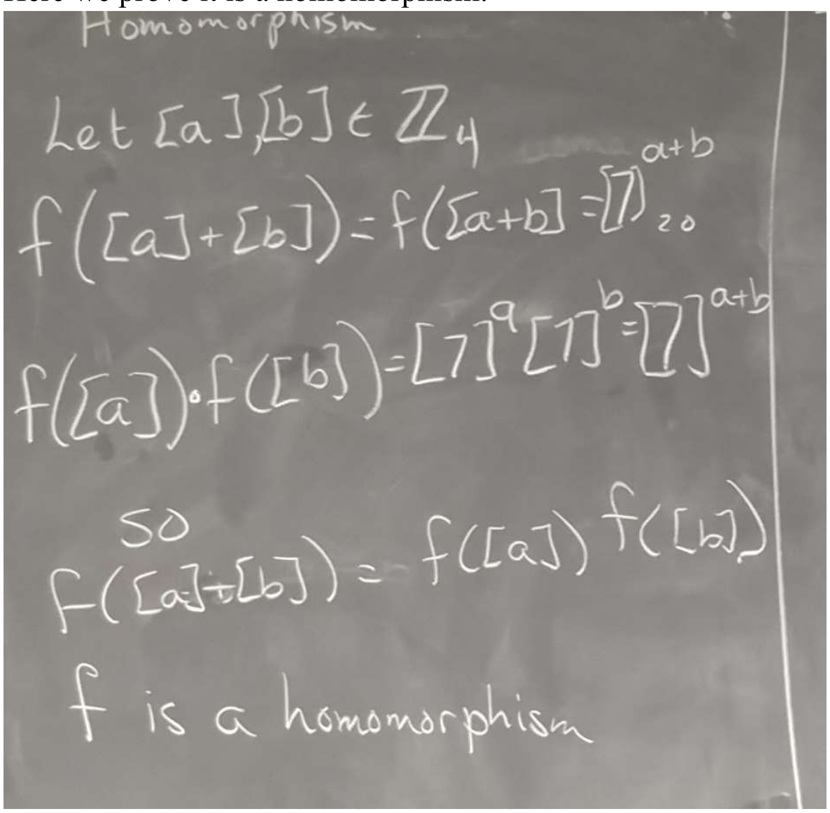
Here we prove it’s a function:



Here we prove this function is 1-to-1




Here we prove it is a homomorphism:




To prove it is onto, we would say:

Let $[a]_{20} \in <7> \subseteq U_{20}$

So $[a] = [7]^n$ for some $n \in \mathbb{Z}$

Then $[n]_4 \in \mathbb{Z}_4$ and $f([n]_4\} = [7]_{20}^n = [a]$

so *f* is onto, and hence *f* is an isomorphism of groups.

The homework is: