## Math 351 Notes (with additional extension) 3-6-2020

Mod numbers are actually sets of integers, In order to be a function on mod numbers, every number in the equivalence class set of a mod number has to be mapped to the same place.

= {2+4n/neZ}  $\begin{bmatrix} 3 \end{bmatrix}_{4} = \{ ..., -5, -1, 3, 7, 11, ... \}$ =  $\{ 3 + 4n \mid n \in \mathbb{Z} \}$ 

The strategy for showing a function is 1-to-1 is very similar to the strategy for showing that a potential function is a function:

To show 1-to-1 to show it's a function. suppose a, b E domain Suppose a, be domain 5.t f(a)= f(b) 5.t. a= b  $d_{0,00}(K)$ 50 A = b50 f is 1-to-1 90 more 10 f(a)=f(b) so f is a function

Here we used that strategy to prove that our first example potential function is really a function.

This is how we show I is a function. Suppose [a],[b]e ] y Such that [a]=[b] so a= m+ 4.4 and b= m+ k.4 where n, y, kell  $f(a) = [2(n+4y)]_8 = [2n+8y]_8 = [2n]_8 + [8y]_8 = [2n]_8 + [8k]_8 = [2n]_8 + [0]_8$  $f(b) = [2(n+4k)]_8 = [2n+8k]_8 = [2n]_8 + [8k]_8 = [2n]_8 + [0]_8$ so f(a) = f(b)50 f is a function.

If it's not a function, this is what happens:

 $f(I_{n+4kJ}) = I_{2n+8kJ_{10}} = I_{2nJ_{10}} + (I_{8jJ_{10}}) + (I_{8kJ_{10}}) + (I_{8kJ$ HW for example E034 = E434 f(10]) = [0],0 f([4],)= [8],0 not equal not a function

Here's most of a proof that something is an isomorphism: This is the definition of the relation we will prove is a function:

 $\frac{f([x]_{4})=[.7^{x}]_{20}}{\langle 7\rangle \leq U_{20}}$ 1-1 Let [a], [b] & Z4 s.t f([a]) = f([b])  $[7]^{3}[7]^{2} = [7]^{2}[7]$  $[a]_{a}=[b]_{a}$ 

Recall from earlier that  $7^2 \equiv 9 \pmod{20}$ ,  $7^3 \equiv 3 \pmod{20}$ ,  $7^4 \equiv 1 \pmod{20}$ 

Here we prove it's a function:

This is how we show I is a function.  $f([b]_{4}) = [7^{n+4k}]_{20} = [7^{n} \cdot 7^{4k}]_{20} = [7^{n}]_{20} = [7^{n}]_{$ 

## Here we prove this function is 1-to-1

 $f([x]_4) = [7]_{20} \rightarrow \langle 7 \rangle \leq U_{20}$ 1 wish a-b=4m for some mETZ  $[a]_{i}=[b]_{i}$ 

## Here we prove it is a homomorphism:

Homomorphism Let [a] [b] E Zu  $f([a]+[b])=f([a+b]=[]_{20}$ ([6])=[7]<sup>9</sup>[7]<sup>b</sup>=[7]<sup>a+b</sup> 50F(LaJt(bJ)) = f(LaJ)f(LbJ)f is a homomorphism

To prove it is onto, we would say: Let  $[a]_{20} \in \langle 7 \rangle \subseteq U_{20}$ So  $[a] = [7]^n$  for some  $n \in \mathbb{Z}$ Then  $[n]_4 \in \mathbb{Z}_4$  and  $f([n]_4) = [7]_{20}^n = [a]$ so f is onto, and hence f is an isomorphism of groups. The homework is:

due Weds 7.4 # 5, 6, 11, 13 HW and problem 3 Prove f: Zg - Zio define d by S(x)=2x is not a function