Abstract Algebra notes and assignment Feb 5 2020

The following sets are groups with the operation addition: +	The following sets are groups with the operation multiplication: $\cdot$
$\mathbb{Z}$ = The integers	$\mathbb{Q}^*$ = All of the rational numbers except 0
$\mathbb{Q}$ = The rational numbers	$\mathbb{R}^*$ = All of the real numbers except 0
$\mathbb{R}$ = The real numbers	C^* = All of the complex numbers except 0
$\mathbb{C}$ = The complex numbers	$GL(2,\mathbb{R})$ = The invertible 2 × 2 matrices with entries that are real numbers. (Invertible means it has a multiplicative inverse).
$M(2,\mathbb{R})$ = 2 × 2 matrices with entries that are real numbers	U_n = The mod-n numbers that are units. (Being a unit means it has a multiplicative inverse)
$\mathbb{Z}_n$ = mod-n numbers	

This week, you should start writing up the proofs of the group theorems. You will be turning these in as a homework assignment in about a week. Of the theorems on the first page of the Definitions and Theorems sheet, you will be responsible for knowing the proofs of theorems 1-5. Some of these proofs will be given in a video #2 and 5), some will be done in class (#1), and some you are responsible for figuring out yourself (3 and 4).

Homework problems:

For each of the following sets and operations, either prove that it is a group (you may use the subgroup theorem: theorem 6) or prove that it is not a group (give a counterexample to show that one of the properties fails).

1. The positive real numbers, with the operation multiplication: $\mathbb{R}^+$
2. The imaginary numbers, with the operation addition: $I = \{ai \mid a \in \mathbb{R}\}$
3. The non-zero imaginary numbers, with the operation multiplication: $I^* = \{ai \mid a \in \mathbb{R}, a \neq 0\}$
4. The diagonal matrices with non-zero real entries, and operation multiplication:
   $D(2,\mathbb{R}^+) = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a, b \in \mathbb{R}^+ \right\}$
5. Matrices of the form: $\left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \middle| a \in \mathbb{R}^+ \right\}$, with operation multiplication. Note that this is *not* a subset of $GL(2,\mathbb{R})$
6. The even numbers, with the operation addition: $2\mathbb{Z} = \{2n \mid n \in \mathbb{Z}\}$
7. The integer powers of 2, with operation multiplication: $\{2^n \mid n \in \mathbb{Z}\}$
8. The odd numbers, with operation addition $\{2n+1 \mid n \in \mathbb{Z}\}$
9. Given a group G with operation written as multiplication that has subgroups H and K is $H \cup K$ a group?
10. Given a group G with operation written as multiplication that has subgroups H and K is $H \cap K$ a group?