Feb 4, 2019

Cor. 7.6
p.197

Theorem 4 If G is a group and $a, b \in G$ then $(ab)^{-1} = b^{-1}a^{-1}$

Theorem 5 If G is a group and $a \in G$ then $(a^{-1})^{-1} = a$

Definition/Notation: If G is a group and $a \in G$ then $a^2 = aa$ and $a^n = \underbrace{aa...a}_{\text{n factors}}$ if n is a positive integer. $a^n = \underbrace{a^{-1}a^{-1}...a^{-1}}_{\text{n factors}}$ if n is a negative integer and $a^0 = e$ where e is the identity. $e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Theorem 6 If G is a group and $a \in G$ then $a^n a^m = a^{n+m}$

HW ① Subgroup of M_2 $\left\langle \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \right\rangle$

② subgroup of S_4 $\langle (124) \rangle$

(Thm 7.7 -not proved in book)
p.198

prove the theorem for the cases:

a) $n = 0$ or $m = 0$
b) $n > 0$ and $m > 0$
c) $n > 0$ and $m < 0$
d) $n < 0$ and $m > 0$
e) $n < 0$ and $m < 0$

Unless you are specifically asked to explain/prove a property, you may assume that the following examples have been proven to be groups:

$\mathbb{R}$ = real numbers (with addition) $\quad$ $\mathbb{Q}$ = rational numbers (with addition)

$\mathbb{C}$ = complex numbers (with addition)

D_n = dihedral group of degree n (symmetries of a regular n-gon, with operation function composition), for integers $n > 3$

S_n = permutation group of degree n = symmetric group of degree n (permutations of n elements, where n is a positive integer

$M_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a, b, c, d \in \mathbb{R} \right\}$ = real valued 2x2 matrices (with addition)

Additionally, you may assume that we know that multiplication is associative for $\mathbb{R}, \mathbb{C}, \mathbb{Q}, \mathbb{Z}$ and M_2 , and multiplication is commutative for $\mathbb{R}, \mathbb{C}, \mathbb{Q}, \mathbb{Z}$

Definition: The order of a group is the number of elements in the group.

Definition: In a group G with element $a \in G$, if $a^n = e$ for some integer $n > 0$, then the element a has finite order. If k is the smallest positive integer such that $a^n = e$, then a has order k. If $a^n \neq e$ for every positive integer n, then a has infinite order.

subgroup generated by a.

Definition: In a group G with elements $a, b \in G$, the set $< a > \subseteq G$ is the smallest subgroup of G that contains a, and $< a, b >$ is the smallest subgroup of G that contains both a and b.