

Some notes about the proof of theorem 56:

1. This is an if and only if theorem, so the proof has two parts.
2. I wrote one of the equations down backwards. Again. I'm pretty sure I made this mistake in the first draft of theorem 19 too. In the second bullet it should be $c(x) \mid d(x)$ and not vice versa.
3. One half of the proof is just like the proof of theorem 19. The other half of the proof should be pretty easy.
4. Hint for one half of the proof: use theorem 55 (you can do that if you know $d(x)$ is a gcd.)
5. Hint for the other half of the proof: if you know $c(x) \mid d(x)$, then you can use theorem 50.
- 6.

Theorem 19 (1.3): Let a and b be integers where not both are zero, and $d = \gcd(a, b)$. Then if $c \mid a$ and $c \mid b$ then $c \mid d$

Proof: Let a and b be integers where not both are zero, and $d = \gcd(a, b)$.

Suppose $c \mid a$ and $c \mid b$

Then there exist $n, m \in \mathbb{Z}$ such that $a = cn$ and $b = cm$.

By theorem 18, there exist $u, v \in \mathbb{Z}$ such that $d = au + bv$

So, $d = (cn)u + (cm)v = c(nu + mv)$

Therefore $c \mid d$