What you should have learned in class today (Feb 8):

- You are allowed to use exponential notation to write products and compositions. In D_4: $r^2 = h$ and $r^3 = l = r^{-1}$
- The order of a group is the number of elements in the group.
- The order of an element is the power that makes it equal to the identity: in D_4 we discovered $r^4 = e$ so *r* has order 4.
- <*r*> is the smallest subgroup of D_4 that includes *r*, so it includes *r*, *e*, $r^2 = h$ and $r^3 = l$. You can write out the composition table for those elements to find that this set is closed and has inverses, so it is a subgroup.
- In any dihedral group, for example D_6 the element *r*, which is the smallest right hand symmetry rotation will have order 6—the same as the degree of the group. The subgroup <*r*> generated by *r* will also have order 6, and will consist of all of the rotations.
- If you are looking for a subgroup of a finite group, like S_4, and you start with a generating element like (123), then what you have to do is start with a table that included *e* and (123) and work out all of the compositions. When you find a new element by composing one of the two you've already got, then you add it to the table and keep expanding until you have a table with no new elements. For example $(123)^2 = (132)$ so you have to include (132) in the subgroup.
- If you are looking for a subgroup of an infinite group like $\mathbb{R}$ or M_2, then you need to start by doing the same thing, but you're going to get an infinite set, so you need to look for patterns so you can identify what all of the elements are.
  - Example 1, in $\mathbb{R}$ if you look for the subgroup $\left\langle \frac{1}{2} \right\rangle$, you will find that you need the numbers 0, ½, -1/2, 1, -1, 3/2, 2, -3/2, -2, … That's going to be all of the possible "halves", so you can write the subgroup $\left\{ \frac{n}{2} \middle| n \in \mathbb{Z} \right\}$. That notation means: the set includes all of the numbers you can get by putting an integer over 2.
  - Example 2: In M_2, if you look for the subgroup $\left\langle \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\rangle$. Since this is an additive group, the identity is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and the elements you need to make a set that is closed and has inverses is $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & n \end{bmatrix} \middle| n \in \mathbb{Z} \right\}$

For Monday, figure out the proofs of theorems 4, 5 and 6 (help available in the textbook), and figure out what the elements need to be for the subgroups $\left\langle \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \right\rangle$ as a subgroup of M_2 (with operation addition) and $<(124)>$ as a subgroup of S_4