April 8, 2019

Problem 26: Let $r \in D_4$ be the counterclockwise rotation by $90°$, and let $S = < r > \subseteq D_4$ be the subgroup generated by *r*. We know from theorem 11 that $S = \{r^n \mid n \in \mathbb{Z}\}, \circ$. You may also assume that you know that r^n is the counterclockwise rotation by $90° \cdot n$

Let $f : S \to \mathbb{Z}_4, +$ such that $f(r^n) = [n]_4$

a. Prove f is a function. To do this you must prove that if $r^n = r^m$ then $[n] = [m]$

b. Prove that *f* is a (group) isomorphism.

Problem 27: Given a ring, *R*, and $a, b \in R$ such that $a \neq 0$

a. Does *a* or *R* need any additional properties in order for us to be able to find a solution for *x* that is an element of *R* to the equation $a + x = b$? (If so, what properties do you need? If not, how would you solve it?)

b. Does *a* or *R* need any additional properties in order for us to be able to find a solution for *x* that is an element of *R* to the equation $ax = b$? (If so, what properties do you need? If not, how would you solve it?)

Problem 28:. I would like to have a ring *R* that has the zero-product property, which states:

If $c, d \in R$ and $cd = 0$ then $c = 0$ or $d = 0$

This property will be true if *R* does not have any elements that are ________________________.

Problem 29:. I would like to have a ring *R* where $x = ba^{-1}$ is the solution to the equation $ax = b$ for all $a, b \in R$ such that $a \neq 0$. What additional properties must *R* have in order for this to be true?

Problem 30: Given a ring *R* with identity. Let $U \subseteq R$ be the set of all units in *R*. Prove that $U, \cdot$ is a group.

Problem 31: U_n is the set of units in $\mathbb{Z}_n$. List the elements in:

a) U_5
b) U_8
c) U_{10}
d) U_{12}
e) U_{15}