April 22, 2019

Notation: we will use the notation T_n to denote the subset of rotations of D_n and we will use r_n to denote the rotation by $360° / n$, so that $T_n = < r_n > \subseteq D_n$

Problem 32: Prove that $T_8 \cong \mathbb{Z}_8, +$

Problem 33: a. Prove that $f : \mathbb{Z}_{12}, + \to \mathbb{Z}_4, +$ such that $f([m]_{12}) = [m]_4$ is a function and a homomorphism (of groups).

b. Why is *f* not an isomorphism?

Problem 34: Note that $U_{14}, \times = \{1, 3, 5, 9, 11, 13\}$ has 6 elements. Compute $< 1 >, < 3 >, < 5 >$ etc. Either find a subgroup that includes all 6 elements or show that no subgroup generated by only one element includes all 6 elements.

Problem 35: $U_9 = \{1, 2, 4, 5, 7, 8\}$. Calculating mod 9 (with multiplication), one gets:

$5^1 \equiv 5, \quad 5^2 \equiv 7, \quad 5^3 \equiv 8, \quad 5^4 \equiv 4, \quad 5^5 \equiv 2, \quad 5^6 \equiv 1$, so $< 5 > = U_9$

Prove that $f : \mathbb{Z}_6, + \to U_9, \times$ such that $f([n]_6) = [5^n]_9$ is a function and an isomorphism.

Problem 36: Define an isomorphism $g : \mathbb{Z}_6, + \to U_{14}, \times$

Problem 37: Find an example that shows that there can be rings $S \subseteq R$ and $T \subseteq R$ that are both subrings of *R*, but $S \cup T$ is **not** a subring of *R*.