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Theorem 39: If $a, b \in R$ then $a(-b) = -(ab)$ and $(-a)b = -(ab)$

Theorem 40: If $a \in R$ then $-(-a) = a$

Theorem 41: If $a, b \in R$ then $-(a+b) = -a + -b$

Theorem 42: If $a, b \in R$ then $(-a)(-b) = ab$

Definition: Saying that ring R , has the **multiplicative cancellation property** means: for $a, b, c \in R$, if $ab = ac$ or $ba = ca$ then $b = c$

Theorem 43: A ring R has the multiplicative cancellation property if and only if R has no zero divisors.

Theorem 44: If $S \subseteq R$ and $T \subseteq R$ are both subrings of R , then $S \cap T$ is a subring of R .

Theorem 45: If R is a ring and $a \in R$ then the set $aR = \{ax \mid x \in R\} \subseteq R$ is a subring of R and the set $Ra = \{xa \mid x \in R\} \subseteq R$ is a subring of R .

Definition: If R and S are rings and $f : R \rightarrow S$ is a function and $a, b \in R$, then f is a ring **homomorphism** if $f(a+b) = f(a) + f(b)$ and $f(ab) = f(a)f(b)$.