Thm 44 an example to think about

$2\mathbb{Z}$ = evens is a ring

$3\mathbb{Z}$ = multiples of 3 is a ring

$2\mathbb{Z} \cap 3\mathbb{Z}$

0, 6, 12, 18, 24, 30

-6, -12, -18, ...

multiples of 6

$= 6\mathbb{Z}$

Let $n, m \in 2\mathbb{Z} \cap 3\mathbb{Z}$

so $n, m \in 2\mathbb{Z}$ and $n, m \in 3\mathbb{Z}$

so $n+m \in 2\mathbb{Z}$ and $n+m \in 3\mathbb{Z}$

so $n+m \in 2\mathbb{Z} \cap 3\mathbb{Z}$

$2\mathbb{Z} \cup 3\mathbb{Z}$

0, 2, 4, 6, 8

-2, -4, -6, -8

3, 9, 15

$2+3 \notin 2\mathbb{Z} \cup 3\mathbb{Z}$

not closed under +.

T 45

If R is a ring and $a \in R$

$aR = \{ax \mid x \in R\}$ is a sub-ring of R

closed $+$: Let $an, am \in aR$
then $an + am = a(n+m) \in aR$

closed $\times$: Let $an, am \in aR$
then $(an)(am) = a(nam)$

has add. inverses: Let $an \in aR$
$-(an) \in R$
$-(an) = a(-n)$ and $-n \in R$
Thm 39 so $a(-n) \in aR$
so $-(an) \in aR$

$2\mathbb{Z} = \{2n \mid n \in \mathbb{Z}\}$ = even integers

$3\mathbb{Z} = \{3n \mid n \in \mathbb{Z}\}$ = multiples of 3

$2\mathbb{Z}_{10} = \{0, 2, 4, 6, 8\}$

Rings

$2\mathbb{Z}_9 = \{0, 2, 4, 6, 8, 1, 3, 5, 7\}$

Ex 2

$$K \subseteq M_2(\mathbb{R})$$

$$K = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \middle| \; a, b \in \mathbb{R} \right\}$$

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -(b+d) & a+c \end{pmatrix} \in K$$

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} ac-bd & ad+bc \\ -bc-ad & -bd+ac \end{pmatrix} \in K$$

$$\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix} \in K$$

$$f : K \longrightarrow \mathbb{C} \qquad f\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = a+bi$$

this is a function

one version input

one version output

Suppose $f\left(\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\right) = f\left(\begin{pmatrix} x & y \\ -y & x \end{pmatrix}\right)$

$$a+bi = x+yi$$

so $a = x$, $b = y$

so $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$ so 1-1

f is onto because ...

Let $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} c & d \\ -d & c \end{pmatrix} \in K$

$$f\left(\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix}\right) =$$

$$f\left(\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\right) + f\left(\begin{pmatrix} c & d \\ -d & c \end{pmatrix}\right) =$$

$$f\left(\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} c & d \\ -d & c \end{pmatrix}\right) =$$

$$f\left(\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\right) \cdot f\left(\begin{pmatrix} c & d \\ -d & c \end{pmatrix}\right) =$$

Ex 1

$$2\mathbb{Z}_{10} = \{0, 2, 4, 6, 8\}$$

$$\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$$

Try the obvious (wrong)

$0 \to 0$
$2 \to 1$
$4 \to 2$
$6 \to 3$
$8 \to 4$

$$f(4 \cdot_{10} 8) = f(32_{10}) = f(2) = 1$$

$$f(4) \cdot f(8) = 2 \cdot 4 = 8_5 = 3$$

$\cdot$ (10)	0	2	4	6	8
0	0	0	0	0	0
2	0	4	8	2	6
4	0	8	6	4	2
6	0	2	4	6	8
8	0	6	2	8	4

6 is $\cdot$ identity

$2\mathbb{Z}_{10}$		$\mathbb{Z}_5$
6	$\longrightarrow$	1

HWK: Prove Thm 44

Prove Ra is a subring (Thm 45)

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