

Discrete math practice problems:

1. Prove by induction that

$$5 + 8 + \dots + (3n + 2) = \frac{n(3n + 7)}{2}$$

2. How many 4 element subsets of {a, b, c, d, e, f} are there? **C(6,4)**

3. There are 90 widgets that need to be assembled by 8 workers. What is the smallest number that the most efficient worker (the one who assembles the most widgets) might assemble? 90 is just 2 more than 88. It's possible for each of them to assemble 11 widgets each, and the most efficient 2 workers assemble 1 more each, so the smallest possible max is **12** widgets by the person who does the most.

4. There are 5 flavors of Jolly Ranchers. If I grab 10 Jolly Ranchers at random out of a bowl, how many different combinations could I get? This is a "donut"-type problem: I'm getting 10 JRs. I'm going to put in 4 dividing lines, separating them into 5 flavors, so there are **C(14,4)** different flavor combinations.

~~What is the probability that I get 10 of the same flavor?~~ This one's actually not as hard as the one we skipped on the homework, but I'm still not putting one like this on the test.

5. I have a stack of 15 different Pokemon cards. 7 are water type and 8 are fire type. Assume each has a different number of HP.

a. In how many ways can I choose 5 cards? **C(15,5)**

b. In how many ways can I choose 3 water type and 2 fire type cards? **C(7,3)C(8,2)**

c. If I choose 5 cards at random, what is the probability that 3 are water type and 2 are fire type?

C(7,3)C(8,2) / C(15,5)

d. If I put down 5 cards, one at a time, how many orders are there? **5!**

e. If I put down 5 cards in a row, what is the probability that the first card has the highest HP? **1/5** (note that it doesn't matter what happens with the other 4 cards)

f. If I put down 5 cards in a row, what is the probability that they are in order of decreasing HP? **1/5!**

6. I am going to give 15 identical Pokemon cards to 4 cub scouts. In how many ways can I do this?

C(18,3) (insert 3 dividing lines to split the cards into 4 groups)

~~What is the probability that each cub scout gets at least 2 cards (if the outcome is completely random)?~~

Nope. Maybe later.

7. I am going to give 15 different Pokemon cards to 4 cub scouts, so scout A gets 6 cards, scout B gets 4 cards, scout C gets 3 cards and scout D gets 2 cards. How many different possible outcomes are there?

C(15,6)C(9,4)C(5,3)C(2,2)

A probability question I could ask here is what is the probability that scout D gets the cards with the two highest HP? The number of ways to do that (because now scout D's cards are determined) is

$C(13,6)C(7,4)C(3,3)$, so the probability is:

$$\frac{C(13,6)C(7,4)C(3,3)}{C(15,6)C(9,4)C(5,3)C(2,2)}$$