

Show/prove that...

Congruence mod 7 is an equivalence relation

Reflexive ($a \equiv a \pmod{7}$):

proof: If a is an integer

$$a - a = 0$$

$$\text{so } a - a = 7 \cdot 0$$

so $a - a$ is a multiple of 7

$$\text{So } a \equiv a \pmod{7}$$

Symmetric (if $a \equiv b \pmod{7}$ then $b \equiv a \pmod{7}$)

proof: if $a \equiv b \pmod{7}$

$$a - b = 7n \text{ for some integer } n$$

$$-1(a - b) = -1(7n)$$

$$-a + b = -7n$$

$$b - a = 7(-n)$$

$$b \equiv a \pmod{7}$$

Transitive (if $a \equiv b \pmod{7}$ and $b \equiv c \pmod{7}$ then $a \equiv c \pmod{7}$)

proof: if $a \equiv b \pmod{7}$ and $b \equiv c \pmod{7}$

so, $a - b = 7n$ and $b - c = 7m$ for some integers n and m

$$\text{so, } a = b + 7n \text{ and } c = b - 7m$$

$$a - c = (b + 7n) - (b - 7m)$$

$$= b + 7n - b + 7m$$

$$= 7n + 7m$$

$$= 7(n + m)$$

$$a \equiv c \pmod{7}$$