Review Practice Problems 9.1-9.3

1. Prove by induction that $5 \cdot 3^n - 3$ is an explicit formula for the recursively defined function:

$S_n = 3S_{n-1} + 6$ where $S_0 = 2$

Check $n = 0$: $5 \cdot 3^0 - 3 = 5 - 3 = 2 = S_0$

Assume: $S_{k-1} = 5 \cdot 3^{k-1} - 3$

Then, $S_k = 3S_{k-1} + 6 = 3(5 \cdot 3^{k-1} - 3) + 6 = 3 \cdot 5 \cdot 3^{k-1} - 9 + 6 = 5 \cdot 3 \cdot 3^{k-1} - 3 = 5 \cdot 3^k - 3$

So, for any $n \geq 0$, $S_k = 5 \cdot 3^n - 3$

2. Use the method of iteration to find an explicit formula for the function:

$S_n = 5S_{n-1} + 3$ where $S_0 = 4$

Make a table:

n	S_n
0	4
1	$5 \cdot 4 + 3$
2	$5(5 \cdot 4 + 3) + 3 = 5^2 \cdot 4 + 5 \cdot 3 + 3$
3	$5(5^2 \cdot 4 + 5 \cdot 3 + 3) + 3 = 5^3 + 5^2 \cdot 3 + 5 \cdot 3 + 3$
4	$5^4 + 5^3 \cdot 3 + 5^2 \cdot 3 + 5 \cdot 3 + 3 = 5^4 + (5^3 + 5^2 + 5 + 1) \cdot 3$
n	$5^n + (5^{n-1} + 5^{n-2} + ... + 1) \cdot 3$

$$S_n = 5^n + (5^{n-1} + 5^{n-2} + ... + 1) \cdot 3 = 5^n + 3 \cdot \frac{5^n - 1}{5 - 1} = 5^n + 3 \cdot \frac{5^n - 1}{4}$$

3. Given that an explicit formula for a linear difference equation will be of the form $A \cdot x^n + B$, find an explicit formula for the function:

$S_n = 7S_{n-1} + 5$ where $S_0 = 3$

$S_1 = 7 \cdot 3 + 5 = 26$

$A \cdot 7^0 + B = 3 \quad \rightarrow -A - B = -3$

$A \cdot 7^1 + B = 26 \quad \rightarrow \underline{7A + B = 26}$

$6A = 23 \qquad A = 23/6$

$23/6 + B = 3$

$$B = 3 - \frac{23}{6} = \frac{18}{6} - \frac{23}{6} = -\frac{5}{6}$$

$$S_n = \frac{23}{6} 7^n - \frac{5}{6}$$