

Set notation and relationships:

Definitions:

- \in "is an element of". $x \in A$ means that x is an element of A . Every set is defined by its elements, and two sets are equal if they have all of the same elements.
- \emptyset "the empty set". The set that contains no elements.
- \subseteq "is a subset of". $A \subseteq B$ if every element of A is also an element of B . Note that Every element of A is an element of A so $A \subseteq A$. Note also that \emptyset has no elements, so it is a subset of every set.
- U "the universe" or "the universal set". The set of all elements that are relevant for the current problem (often the set of all numbers or all points in the plane).
- $|$ "the number of elements of". $|A|$ is the number of elements in A . Note that $|\emptyset| = 0$ because the empty set does not have any elements.
- \cap "intersection". The intersection of two sets is the set of all of the elements that are in both sets: $x \in A \cap B$ if $x \in A$ and $x \in B$ both
- \cup "union". The union of two sets is the set of all of the elements that are in either set: $x \in A \cup B$ if $x \in A$ or $x \in B$ (or both—"or" when used about sets is always inclusive: one or the other or both).
- $\overline{}$ "complement". The complement of a set is all of the elements in the Universe that are not in the set: $x \in \overline{A}$ if $x \notin A$
- $-$ "minus". $A - B$ is all of the elements that are in A that are not in B : $x \in A - B$ if $x \in A$ but $x \notin B$

Theorems/relationships:

Given sets A, B, C in universal set U

<p>Thm 2.0</p> <ul style="list-style-type: none"> a. $A \cup U = U$ b. $A \cap U = A$ c. $A \cup \emptyset = A$ d. $A \cap \emptyset = \emptyset$ e. $A \cup A = A$ f. $A \cap A = A$ 	<p>Thm. 2.1</p> <ul style="list-style-type: none"> a. $A \cup B = B \cup A$ and $A \cap B = B \cap A$ b. $A \cup (B \cap C) = (A \cup B) \cap C$ and $A \cap (B \cup C) = (A \cap B) \cup C$ c. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ d. $\overline{\overline{A}} = A$ (that's a double complement, not an =) e. $A \cup \overline{A} = U$ f. $A \cap \overline{A} = \emptyset$ g. $A \subseteq A \cup B$ and $B \subseteq A \cup B$ h. $A \cap B \subseteq A$ and $A \cap B \subseteq B$ i. $A - B = A \cap \overline{B}$ 	<p>Thm 2.2 (DeMorgan's laws)</p> <ul style="list-style-type: none"> a. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ b. $\overline{A \cap B} = \overline{A} \cup \overline{B}$
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18. Show $(A - B) \cup (A \cap B) = A$ using theorems and by making Venn diagrams.

Steps	Theorem	
$(A - B) \cup (A \cap B)$		<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>$A - B$</p> </div> <div style="text-align: center;"> <p>$A \cap B$</p> </div> </div>
$= (A \cap \overline{B}) \cup (A \cap B)$		
$= A \cap (\overline{B} \cup B)$		
$= A \cap U$		
$= A$		

19. Show $(A - B) \cap (A \cup B) = A - B$ using theorems and by making Venn diagrams.

Steps	Theorem	
$(A - B) \cap (A \cup B)$		
=	2.1.i	
=	2.1.b (intersection)	
=	2.1.c (second version)	
=	2.1.f	
=	2.0.c	
=	2.1.a	
=	2.1.b	
=	2.0.f	
=	2.1.i	

Homework: Prove each of the following by using theorems and making Venn diagrams

17. $A \cap (B - A) = A \cap B$

21. $\overline{A} \cap (A \cup B) = B - A$

22. $\overline{(A - B)} \cap A = B \cap A$

(hint: step 1 uses thm 2.1.i and step 2 uses thm 2.2.b)