Induction for subsets:

To prove: Given a set S containing n elements, $n \geq 1$ and $0 \leq r \leq n$ then the number of subsets of S with exactly r elements is $C(n,r) = \dfrac{n!}{r!(n-r)!}$

Step 1: check the case when $n=1$. How many 0-element subsets are there? How many 1-element subsets are there? What does the formula say for these cases?

$S = \{a\}$ has 2 subsets: $\{a\}$, $\emptyset$

$\{a\}$: 1 element; $\emptyset$: 0 elements

$C(1,1) = \dfrac{1!}{1!0!} = 1$ set with 1 element

$C(1,0) = \dfrac{1!}{0!1!} = 1$ set with 0 elements

Step 2: write the induction hypothesis (we will be doing induction on *n*.)

Assume if S has n elements and $0 \leq r \leq n$ then there are $C(n,r)$ subsets with r elements

if $1 \leq n \leq k$

Step 2.5: write the k+1 version on a sticky note.

Step 3: Write out the more complicated (in words) side of the statement from the sticky note. How can you use the induction hypothesis to take care of some of the subsets?

Let S be a set with k+1 elements

Let $0 \leq r \leq k+1$

If I have a subset with r elements

either it includes the last element or it only includes first k elements

$$C(k, r-1) + C(k,r)$$

$$\frac{k!}{(r-1)!(k-(r-1))!} + \frac{k!}{r!(k-r)!}$$

$$\frac{r \cdot k!}{\underbrace{r(r-1)!}_{r!}\,(k-r+1)!} + \frac{k!\,(k-r+1)}{r!\,\underbrace{(k-r)!\,(k-r+1)}_{(k-r+1)!}}$$

common factor

$$= \frac{r\,k! + (k-r+1)\,k!}{r!\,(k-r+1)!}$$

If S has k+1 elements and $0 \le r \le k+1$ then ~~the~~ there are $C(k+1, r)$ subsets with r elements $= \frac{(k+1)!}{r!(k+1-r)!}$

$$= \frac{(\cancel{r} + k - \cancel{r} + 1)k!}{r!(k+1-r)!}$$

$$= \frac{(k+1)k!}{r!(k+1-r)!}$$

$$= \frac{(k+1)!}{r!(k+1-r)!} = C(k+1, r)$$

So by induction, the formula always works for $n \ge r \ge 0$

$S = \{a, e, i, o, u\}$

How many 4-element subsets?

$$C(5,4) = \frac{5!}{4!\,1!} = 5$$

How many subsets total are there? $2^5 = 32$

How many 4 letter "words" are there?

$$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} = 120 = P(5,4) = \frac{5!}{(5-4)!}$$

With dinner you get 2 sides, out of 9 possible $C(9,2)$

subsets of size 2.

If you can get 2 of the same side, then $C(9,2) + 9$

2.6 # 13, 17, 19, 21, 25

8.3 # 13 – 16