$\text{prove } p \to q$ direct proof

Ex. A.9 If $\overbrace{n \text{ is even}}^{p}$ then $\overbrace{n^2 \text{ is even}}^{q}$

$n = 2k$

$\underbrace{\text{if } \overbrace{n \text{ is even}}^{p}}$ $\to$ $\underbrace{n = 2k \text{ for some integer } k}_{r}$

$n^2 = (2k)^2 = 4k^2$

$= 2(2k^2)$

$\boxed{p \to r \text{ is definition}}$

$\underbrace{n = 2k}_{r} \longrightarrow \underbrace{n^2 = 2(2k^2)}_{s}$

$\boxed{r \to s \text{ true by algebra}}$

$(p \to r) \land (r \to s) \land (s \to q)$

$\longrightarrow$

$p \to q$

$\underbrace{n^2 = 2(2k^2)}_{s} \to$ so $\underbrace{n^2 \text{ is even}}_{q}$

$\boxed{s \to q \text{ true by definition}}$

Ex. A.11 If $\underbrace{x + y > 100}_{p}$ then $\underbrace{\underbrace{x > 50}_{a} \text{ or } \underbrace{y > 50}_{b}}_{q}$

$q = a \vee b$

$\sim q = \sim(a \vee b)$

$= \sim a \wedge \sim b$ (de Morgan)

Suppose $x \le 50$ and $y \le 50$

So $x + y \le 100$ (algebra)

Suppose $\sim q$

$\sim p$

have proved $\sim q \to \sim p$

Logically equivalent to

$p \to q$

So if $x+y > 100$

then $x > 50$ or $y > 50$

proof by contrapositive

Ex. A.12 If n^2 is even, then n is even.

If $\underbrace{\text{n is a multiple of 6}}_{p}$, then $\underbrace{n^2 \text{ is even}}_{q}$,

$n = 6k$

$n^2 = (6k)^2 = 36k^2 = 2(18k^2)$

n^2 is even

$p \to r$ r: "$n = 6k$"
true by defn

$r \to s$. s: "$n^2 = 2(18k^2)$"
true by algebra

$s \to q$
true by defn

If $\underbrace{x+y>100}_{p}$ then $\underbrace{\underbrace{x>50}_{a} \text{ or } \underbrace{y>50}_{b}}_{q}$

note

$\sim q = \sim(a \vee b)$

$= \sim a \wedge \sim b$

Given $x+y>100$

Given p

Suppose $x \le 50$ and $y \le 50$

suppose $\sim q$

$(p \wedge \sim q)$

$\downarrow$

is False

then $x+y \le 100$

$x+y \le 100$ and $x+y>100$ is a contradiction

$\downarrow$

$\sim(p \wedge \sim q)$ is true

$\to$ So, if $x+y>100$ then $x>50$ and $y>50$

Logically equiv to $p \to q$

If $\underbrace{x+y>100}_{p}$ then $\underbrace{\underbrace{x>50}_{a} \text{ or } \underbrace{y>50}_{b}}_{q = a \vee b}$

proof using contrapositive:

Suppose $\sim q = \sim(a \vee b) = \sim a \wedge \sim b$

$x \le 50$ and $y \le 50$

then $x+y \le 100$

so $\sim p$

This proves $\sim q \rightarrow \sim p$ which is logically equiv to

$p \rightarrow q$

so $p \rightarrow q$ $\square$

If $\underbrace{x+y>100}_{p}$ then $\underbrace{\underbrace{x>50}_{a} \text{ or } \underbrace{y>50}_{b}}_{q}$

(note $\sim q = \sim(a \vee b) = \sim a \wedge \sim b$)

proof by contradiction:

given $x+y>100$ (p)

suppose $x \le 50$ and $y \le 50$ $(\sim q)$

(so far we have assumed $p \wedge (\sim q)$)

so $x+y \le 50+50=100$, but $x+y>100$, so $x+y>x+y$ which is false

(so we have proved $\sim(p \wedge \sim q)$)

Thus $x>50$ or $y>50$ (q) $\quad p \to q$ is LE to $\sim(p \wedge \sim q)$

If $\underbrace{x+y>100}_{p}$ then $\underbrace{x>50}_{a}$ or $\underbrace{y>50}_{b}$

another proof

given $x+y>100$ (p)

suppose $x \le 50$ $(\sim a)$

then $x+y \le 50+y$ (algebra)

so $50+y>100$ (transitive)

so $y>100-50$

$y>50$ (b) (algebra)

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this says $(p \wedge \sim a) \to b$

show this is LE to $p \to a \vee b$

12.

If n^2 is even then n is even

prove by contradiction

given: n^2 is even $\quad p$

suppose ($\sim q$) n is not even

so n is odd

$n = 2k+1$ for some integer k

$n^2 = (2k+1)^2 = 2^2k^2 + 2\cdot 2k + 1$

$= 2(2k^2+2k)+1$ is odd

~~so $\sim p$~~ which contradicts p

so $\sim(p \vee \sim q)$

so $p \to q$

13.

Given n is sum of sq. of odds
then n is not a perfect square
do by contradiction

(Given p) suppose n is the sum of odd integers
$2a+1$ and $2b+1$

(suppose $\sim q$) suppose n is the square of integer k.

then $(2a+1)^2 + (2b+1)^2 = k^2$

$4a^2 + 4a + 1 + 4b^2 + 4b + 1 = k^2$

$2(2a^2 + 2a + 2b^2 + 2b + 1) = k^2$

2 odd

~~not~~ False

so $\sim(p \wedge \sim q)$ is true

so $p \Rightarrow q$. is true