Set notation and relationships:

Definitions:

$\in$ "is an element of". $x\in A$ means that x is an element of A. Every set is defined by its elements, and two sets are equal if they have all of the same elements.

ϕ "the empty set". The set that contains no elements.

$\subseteq$ "is a subset of". $A\subseteq B$ if every element of A is also an element of B. Note that Every element of A is an element of A so $A\subseteq A$. Note also that ϕ has no elements, so it is a subset of every set.

U "the universe" or "the universal set". The set of all elements that are relevant for the current problem (often the set of all numbers or all points in the plane).

$|\ |$ "the number of elements of". $|A|$ is the number of elements in A. Note that $|\phi|=0$ because the empty set does not have any elements.

$\cap$ "intersection". The intersection of two sets is the set of all of the elements that are in both sets: $x\in A\cap B$ if $x\in A$ and $x\in B$ both

$\cup$ "union". The union of two sets is the set of all of the elements that are in either set: $x\in A\cup B$ if $x\in A$ or $x\in B$ (or both—"or" when used about sets is always inclusive: one or the other or both).

$\overline{}$ "complement". The complement of a set is all of the elements in the Universe that are not in the set: $x\in\overline{A}$ if $x\notin A$

$-$ "minus". $A-B$ is all of the elements that are in A that are not in B: $x\in A-B$ if $x\in A$ but $x\notin B$

Theorems/relationships:

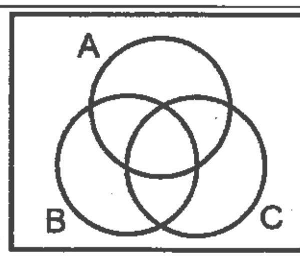
Given sets A,B,C in universal set U

Thm 2.0	Thm. 2.1	Thm 2.2 (DeMorgan's laws)
a. $A\cup U=U$ b. $A\cap U=A$ c. $A\cup\phi=A$ d. $A\cap\phi=\phi$ e. $A\cup A=A$ f. $A\cap A=A$	a. $A\cup B=B\cup A$ and $A\cap B=B\cap A$ b. $A\cup(B\cup C)=(A\cup B)\cup C$ and $\boxed{A\cap(B\cap C)=(A\cap B)\cap C}$ c. $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ and $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ d. $\overline{\overline{A}}=A$ (that's a double complement, not an =) e. $A\cup\overline{A}=U$ f. $A\cap\overline{A}=\phi$ g. $A\subseteq A\cup B$ and $B\subseteq A\cup B$ h. $A\cap B\subseteq A$ and $A\cap B\subseteq B$ i. $A-B=A\cap\overline{B}$	1 2 a. $\overline{A\cup B}=\overline{A}\cap\overline{B}$ $\leftarrow$ b. $\overline{A\cap B}=\overline{A}\cup\overline{B}$ $\leftarrow$ 3. 4

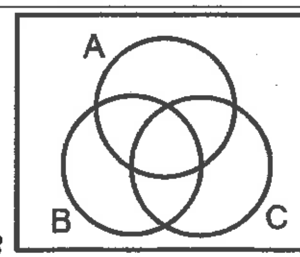
18. Show $(A-B)\cup(A\cap B)=A$ using theorems and by making Venn diagrams. $(A-B)\cup(A\cap B)$

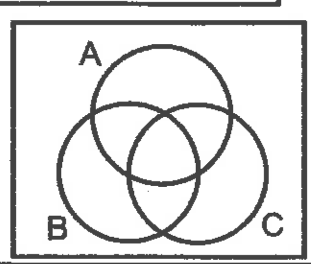
Steps	Theorem
$(A-B)\cup(A\cap B)$	~~2.1.c~~
$=(A\cap\overline{B})\cup(A\cap B)$	2.1.i
$=A\cap(\overline{B}\cup B)$	2.1.c
$=A\cap U$	2.1.e
$=A$	2.0b

$A-B$



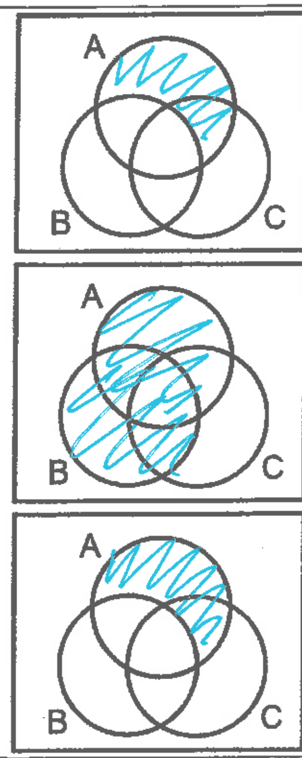
$A\cap B$





19. Show $(A-B)\cap(A\cup B) = A-B$ using theorems and by making Venn diagrams.

	Steps	Theorem
	$(A-B)\cap(A\cup B)$	
	$=(A\cap\overline{B})\cap(A\cup B)$	2.1.i
2	$=A\cap(\overline{B}\cap(A\cup B))$	2.1.b (intersection)
3	$=A\cap((\overline{B}\cap A)\cup(\overline{B}\cap B))$	2.1.c (second version)
4	$=A\cap((\overline{B}\cap A)\cup\emptyset)$	2.1.f
5	$=A\cap(\overline{B}\cap A)$	2.0.c
6	$=A\cap(A\cap\overline{B})$	2.1.a
7	$=(A\cap A)\cap\overline{B}$	2.1.b
8	$=A\cap\overline{B}$	2.0.f
9	$=A-B$	2.1.i



$A-B$

$A\cup B$

$(A-B)\cap(A\cup B$

Homework: Prove each of the following by using theorems and making Venn diagrams

17. $A\cap(B-A)=A\cap B$

21. $\overline{A}\cap(A\cup B)=B-A$

22. $\overline{(A-B)}\cap A=B\cap A$

(hint: step 1 uses thm 2.1.i and step 2 uses thm 2.2.b)