

Ex 1

Let S be the set of integers

$x R y$ means $x - y$ is a multiple of 3

$$Z = 7$$

Reflexive

$$x \sim x$$

$$x R x$$

$$x - x = 0$$

set up condition

$$= 0 \cdot 3$$

← show it works

Symmetric: If $x R y$

$$\exists n (x - y = 3 \cdot n) \quad (1)$$

n is an integer

$$y - x = -3 \cdot n$$

$$y - x = 3(-n)$$

So $y R x$

Transitive: If $x R y$ and $y R z$

$$x - y = 3n$$

$$y - z = 3m$$

$$x = y + 3n$$

$$y = x - 3n$$

$$y = 3m + z$$

$$z = y - 3m$$

$$x - z = y + 3n - (y - 3m) = 3n + 3m = 3(n + m) \quad \therefore$$

OR

$$x - 3n = 3m + z$$

$$x - z - 3n = 3m$$

$$x - z = 3m + 3n$$

$$3(m + n)$$

so $x R z$

Ex 1.

$$7 \mathbb{R} \frac{x}{\downarrow}$$

$$4$$

$$7-4 = 1 \cdot 3$$

$$1$$

$$7-1 = 2 \cdot 3$$

$$-2$$

$$7-(-2) = 3 \cdot 3$$

$$-5$$

$$7-(-5) = 4 \cdot 3$$

$$\textcircled{7}$$

$$7-7 = 0 \cdot 3$$

$$10$$

$$7-10 = -1 \cdot 3$$

$$7-x = 3 \cdot n$$

$$x = 7 - 3n$$

$$[7] = \{ x \mid x = 7 - 3n \text{ for some integer } n \}$$

7-x is a multiple of 3

$S =$ ordered pairs of real numbers $z = (5, 4)$

$(x_1, x_2) R (y_1, y_2)$ means $x_1 - y_2 = y_1 - x_2$

$(a, b) R (c, d)$ means $a - d = c - b$

means
 $a + b = c + d$

Reflexive: If $(x, y) \in S \leftarrow$

then $x + y = x + y$

$(x, y) R (x, y)$

Symmetric: If $(x, y) R (n, m)$

$x + y = n + m$

so $n + m = x + y$

so $(n, m) R (x, y)$

Transitive: If $(x, y) R (a, b)$ and $(a, b) R (n, m)$

$x + y = a + b$

$a + b = n + m$

so $x + y = n + m$

thus $(x, y) R (n, m)$

$(5,4) \sim (a,b)$ mean

$$5 - b = a - 4$$

$$(a,b) = (8,1)$$

$$(1,8)$$

$$(5,4)$$

$$(2,7)$$

$$(10,-1)$$

$$(6,3)$$

$$5+4 = a+b$$

$$[(5,4)] = \{ (8,1), (1,8), (6,3) \dots \}$$

$$\{ (x,y) \mid x+y = 9 \}$$