1. $f: \mathbb{R}\times\mathbb{R} \to \mathbb{R}\times\mathbb{R}$ $\quad f(x,y) = (-y, x)$

a. $f(3,5) = (-5, 3)$

b. $\{(x,y) \mid y = 2x\}$

$= \{(x, 2x)\}$ $\quad y = 2x$

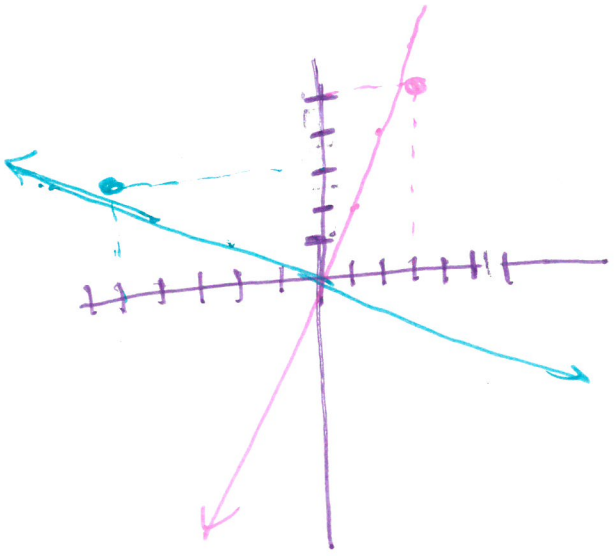
$f(\{x, 2x\}) = \{(-2x, x)\}$

$(-2a, a)$

$\downarrow$

$x = -2a \quad y = a$

$\frac{x}{-2} = a \to y = -\frac{x}{2}$



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c. $f(x,y) = (-y, x) = (2, 6)$

$x = 6, \; y = -2$

$f^{-1}(2,6) = (6, -2)$

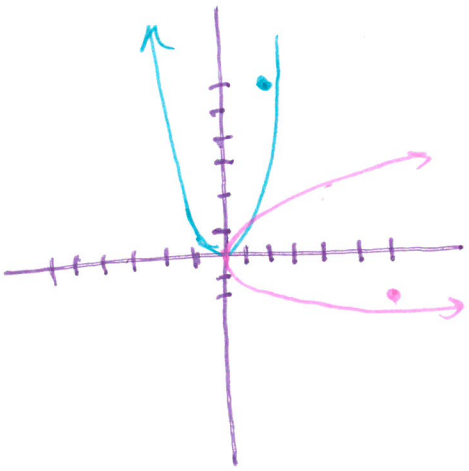
d. $\{(x,y) \mid y = x^2\} = \{(a, a^2)\}$

$f(x,y) = (-y, x) = (a, a^2)$

$\left.\begin{array}{l} x = a^2 \\ -y = a \\ y = -a \end{array}\right\} \to x = (-y)^2 = y^2$

$f^{-1}\{(a, a^2)\} = \{(a^2, -a)\}$

$= \{(x,y) \mid x = y^2\}$



2. $f: \mathbb{R}^2 \to \mathbb{R}^2$ such that $f(x,y) = (x+3, y-1)$

a. $f(4,2) = (7,1)$

b. $\{(x,y) \mid y = |x|\} = \{(a, |a|)\}$

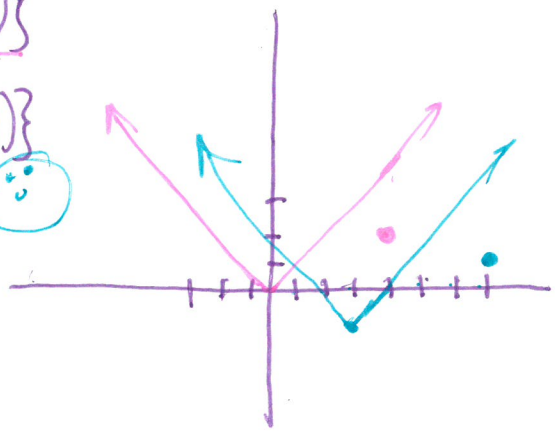
$f(\{(a,|a|)\}) = \{(a+3, |a|-1)\}$

Change form of eqn

$x = a+3 \quad a = x-3$

$y = |a| - 1$

$y = |x-3| - 1$

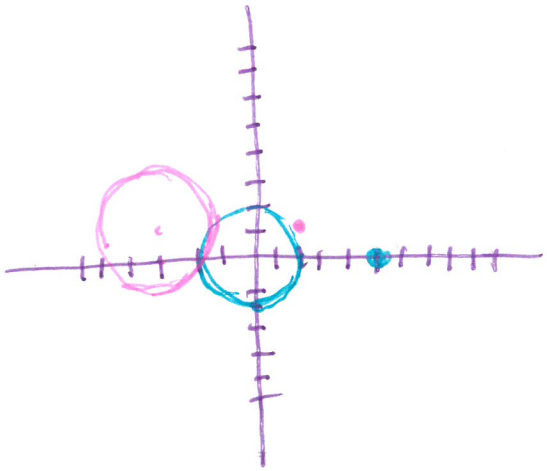


c. $f(x,y) = (x+3, y-1) = (5,0)$

$x+3 = 5 \quad y-1 = 0$

$x = 2 \quad y = 1$

$f(5,0) = (2,1)$



d. $\{(x,y) \mid x^2+y^2 = 4\}$

$f(a,b) = (a+3, b-1) = (x,y)$

$(a+3)^2 + (b-1)^2 = 4$

$f(\{(x,y) \mid x^2+y^2 = 4\})$

$= \{(x,y) \mid (x+3)^2 + (y-1)^2 = 4\}$

3. $g : \mathbb{Z} \to \mathbb{Z}$ such that $g(x) = 4x$

a. $g(10) = 40$

b. $g(3\mathbb{Z}) = \{4 \cdot 3n \mid n \in \mathbb{Z}\} = \{12n \mid n \in \mathbb{Z}\} = 12\mathbb{Z}$

notice notation

c. $g^{-1}(8) = 2$ $\quad -1$: pre.-image

d. $g^{-1}(10) = \emptyset$

e. $g^{-1}(3\mathbb{Z}) = g^{-1}(\{\ldots -12, -9, -6, -3, 0, 3, 6, 9, 12, \ldots\})$

$= \{\ldots -3, 0, 3, \ldots\} = 3\mathbb{Z}$

4. $h: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ such that $h(x,y) = xy$

a. $h(2,3) = 2 \cdot 3 = 6$

b. $h(\{(2,x) \mid x \in \mathbb{Z}\}) = \{2 \cdot x \mid x \in \mathbb{Z}\} = \underbrace{2\mathbb{Z}}$

notice notation

c. $h^{-1}(4)$

$h(x,y) = 4$

$x \cdot y = 4$ $\quad 1 \cdot 4 \quad 2 \cdot 2 \quad -1 \cdot -4 \quad -2 \cdot -2$

$h^{-1}(4) = \{(1,4), (4,1), (2,2), (-1,-4), (-4,-1), (-2,-2)\}$

d. $h^{-1}(3\mathbb{Z}) =$.

· $x \cdot y = 3 \not\in \mathbb{N}$ means x is divisible by 3 or y is divisible by 3

$h^{-1}(3\mathbb{Z}) = \{(3n, m), (n, 3m) \mid n, m \in \mathbb{Z}\}$