Sets part 1

A set is a collection of objects, called elements. These objects can be numbers, points, shapes or even functions.

Every set is defined by its elements: if a set has exactly the same elements as another set, then they are the same set. The elements in a set do not have an order (they can be ordered, but that isn't part of being a set).

In algebra, we work with sets of numbers, for example, the interval (0,1] is the set $\{x \mid 0 < x \leq 1\}$. Many concepts in higher mathematics are described using sets.

💻 If you have done object oriented programming, you can thing of elements as being objects in the programming sense.

In programming, the usual way to have a collection of objects is to put them into an array. Most arrays have are indexed by numbers, so they have an order. If you only ever accessed your array objects by an un-ordered key, that would make the array more like a set. Sets are different from arrays in that they can have infinitely many elements.

Defining a set is like defining a class of objects—all possible objects of that class make up the set.

Commonly used sets and notation (learn these symbols)

$\mathbb{R}$ is the set of real numbers

$\mathbb{N} = \{1, 2, 3, \ldots\}$ is the set of counting numbers

$\mathbb{Z} = \{\ldots, -2, -1, 0, -, 2, \ldots\} = \{0, 1, -1, 2, -2, \ldots\}$ is the set of integers

$\mathbb{Q} = \left\{ \frac{a}{b} \middle| a \in \mathbb{Z}, b \in \mathbb{N} \right\}$ is the set of rational numbers

$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$ is the set of complex numbers

Recall that the things in the brackets { } describe what is in the set. If there is something before a | symbol, then that shows what form the elements have. If there is something after that |, that tells the properties of the variables.

Ordered pairs/triples/etc.

Often we want the elements of the set to have more information than just a number. For example, if we want a point in the $x, y-\text{plane}$, we need 2 coordinates. We use the × symbol to show that we will have 2 real-number coordinates like this: $\mathbb{R} \times \mathbb{R} = \{(x, y) \mid x, y \in \mathbb{R}\}$.

Elements and subsets: Elements are the things sets are made of. Subsets are other sets with the property that all of the elements in the subset are also in the larger set.

Symbols	Correct			Not correct
$\in$ means "is an element of"	$2 \in \mathbb{Z}$	$-2, 0 \in \mathbb{Z}$	$\{2\} \notin \mathbb{Z}$	$\{2\} \in \mathbb{Z}$
$\notin$ means "is not an element of	$.5 \notin \mathbb{Z}$	$\{1, 2, 3\} \subseteq \mathbb{Z}$	$\{\} \subseteq \mathbb{Z}$	$2 \subseteq \mathbb{Z}$
$\subseteq$ means is a subset of	$\{2\} \subseteq \mathbb{Z}$	$\mathbb{Z} \subseteq \mathbb{Z}$	$\mathbb{Z} \subseteq \mathbb{R}$	$\mathbb{R} \subseteq \mathbb{Z}$

More examples of sets

$4\mathbb{Z} = \{4n \mid n \in \mathbb{Z}\}$ is the multiples of 4 $0, 4, 8, -4 \in 4\mathbb{Z}$ and $1, 2, 3 \notin 4\mathbb{Z}$	L is the set of all lines in $\mathbb{R} \times \mathbb{R}$ $\{(x, y) \mid y = 3x - 5\} \in L$ and $(2,1) \notin L$
$TP = \{\Delta(a,b)(c,d)(e,f) \mid a,b,c,d,e,f \in \mathbb{R}\}$ is the set of all triangles with specified vertices in $\mathbb{R} \times \mathbb{R}$ The triangle with vertices (0,0), (1,1), (0,3) = $\Delta(0,0)(1,1)(0,3)$ is an element of TP. The points (0,0), (1,1), (0,3) are not elements of TP, and a general equilateral triangle (no particular position) is not an element of TP	

The power set is the set of all subsets of a set and is written 2^S (where S is the name of the set)

Example: $2^{\mathbb{R}}$ is the set of all subsets of $\mathbb{R}$, so $\mathbb{Z} \in 2^{\mathbb{R}}$ and $\{3\} \in 2^{\mathbb{R}}$ but $\mathbb{Z} \not\subseteq 2^{\mathbb{R}}$ and $3 \not\subseteq 2^{\mathbb{R}}$

Set Practice Problems

For 1-6:

- Circle each of the things (a-f) that are elements of the set.
- Draw a line under each thing that is a subset of the set
- Write one more example of an element of the set.

1. $\mathbb{Z}\times\mathbb{Q}$

a. 3 b. $(3,3)$ c. $\left(3,\frac{1}{2}\right)$ d. $\left(\frac{1}{2},\frac{1}{2}\right)$ e. $\{(1,2),(2,0.5),(1,-3)\}$ f. $\{(x,y)\mid x=3, y\in\mathbb{Z}\}$

2. $2^{\mathbb{Q}}$

a. 3 b. $\{3\}$ c. $\{\}$ d. $\mathbb{Z}$ e. $\left\{\frac{1}{n}\middle| n\in\mathbb{N}\right\}$ f. $\{\{x,-x\}\mid x\in\mathbb{Z}\}$

3. $2^{\mathbb{R}\times\mathbb{R}}$

a. $\sqrt{2}$ b. $(\sqrt{2},\sqrt{2})$ c. $\{(\sqrt{2},\sqrt{2})\}$ d. $\{\{(\sqrt{2},\sqrt{2})\}\}$

e. $\{(a,b)\mid a,b\in\mathbb{Z}\}$ f. $\{\{(x,y)\mid x,y\in\mathbb{R}, x^2+y^2=r^2\}\mid r\in\mathbb{R}\}$

4. $L=\{\{(x,y)\mid ax+by=c, x\in\mathbb{R}, y\in\mathbb{R}\}\mid a,b,c\in\mathbb{R}\}$ = the set of lines in the plane

a. $\{(0,y)\mid y\in\mathbb{R}\}$ b. $\{y\mid y=0\}$ c. $\{(x,y)\mid y=3x\}$ d. $\{\{(x,y)\mid x\in\mathbb{R}\}\mid y\in\mathbb{R}\}$

e. $\{\{(x,y)\mid x,y\in\mathbb{R}, y=mx\}\mid m\in\mathbb{R}\}$ f. $\mathbb{R}\times\mathbb{R}$

5. $C^1(\mathbb{R})=\{f:\mathbb{R}\to\mathbb{R}\mid f \text{ is differentiable}\}$

a. $f(x)=3$ b. $f(x)=x^2+4x$ c. $f(x)=\frac{1}{x}$ d. $\{f(x)=mx\mid m\in\mathbb{R}\}$

e. $f(x)=\sqrt{x}$ f. L from problem 4

6. $S^1=\{(x,y)\mid x,y\in\mathbb{R}, x^2+y^2=1\}$

a. $(1,0)$ b. $\{(0,1)\}$ c. $\{(\sin x,\cos x)\mid x\in\mathbb{R}\}$ d. $\{(|\sin x|,\cos x)\mid x\in\mathbb{R}\}$ e.(0,0) f. $\{\}$

For 7-9, write an element in each set:

7. $\mathbb{R}\times\mathbb{R}\times\mathbb{R}$ 8. $C^1(\mathbb{R})\times\mathbb{R}$ (see #5) 9. $S=\{\{(x,y)\mid x,y\in\mathbb{R}, x^2+y^2=r^2\}\mid r\in\mathbb{R}\}$