

Practice with negation, contrapositive and proof:

1. Write the negation of each of these statements.

- The numbers is a multiple of 6 and multiple of 9
- The triangle is isosceles or acute.
- The line has a positive y-intercept and is not vertical.
- All of the numbers in the set are prime
- None of the triangles in the set are obtuse.

2. Write the contrapositive of each if-then statement:

- If a number in the set is prime, then it is odd
- If a quadrilateral in the set has 2 right angles then it does not have 4 equal sides
- If point in the set has a positive x coordinate and a positive y coordinate then it lies inside the unit circle.
- If a function in the set is a parabola, then the vertex has a positive x-coordinate or a positive y-coordinate.

3. Show that this is a tautology:

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

Prove each of these statements using algebra assuming that a, b and c are all integers

- If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$
- The sum of two even integers is even
- The product of two odd integers is odd
- If a divides b and b divides c then a divides c
- If a divides b and a divides $b+2$ then $a = \pm 1$ or $a = \pm 2$

In class examples:

- The product of an even integer and an odd integer is even
- If a divides b and a divides c then a divides $bk + cj$ for any integers j and k
- If $a \equiv c \pmod{n}$ and $b \equiv d \pmod{n}$ then $a + b \equiv c + d \pmod{n}$

Definitions and formulas:

$x \equiv y \pmod{n}$ means that $x = y + kn$
(where k is some integer)

An integer n is even means $n = 2k$ for some integer k

An integer n is odd means $n = 2k + 1$ for some integer k

n divides m means that n divides evenly into m , and n is a factor of m .

Algebraically it means that $m = nk$ for some integer k