

A list of different kinds of counting and probability problems:

- I. How many subsets (total or of size r : 2^n or $C(n, r)$
 - a. Forming committees
 - b. Choosing a group of a given size
- II. What is the smallest size possible for the biggest of m subsets of n objects? $\lceil n/m \rceil$
 - a. Pigeonhole principle
 - b. Distributing a total to m groups
 - c. Selecting enough total of m types to get a minimum number of the same type
- III. Ordering elements ($P(n, r)$ or multiplication)
 - a. How many orders of a group of distinguishable objects
 - b. How many arrangements of a list or seating chart
- IV. Selection with replacement, ordered (multiplication)
 - a. How many phone numbers?
 - b. How many license plates?
- V. Ordering with duplicates? (multiplication, divided by permutations of duplicates)
 - a. How many distinguishable arrangements of PARROT?
- VI. Orders with conditions (multiplication)
 - a. Seating order if the first 2 are F and the last 3 are male?
 - b. Seating order if the first 2 are F or the last 3 are male?
 - c. How many ordered sets of 8 cards if the first card is a heart and the last 2 cards are diamonds?
- VII. Choosing a total amount from different types of identical objects ($C(n+t-1, t-1) = C(n+t-1, n)$)
 - a. Boxes of donuts
 - b. Bags of candy/cookies/bagels
 - c. Boxes of T-shirts of colors/sizes
- VIII. Probabilities (most of the above, but generally not VII)
 - a. Subsets (I): probability that 3 counts are heads, or a committee has 2 F
 - b. Orders with conditions (probability that first card is a heart, or all cards are in order)
 - c. Orders with duplicates (probability that the repeated letters are together)

Answers to review problems

24. a. 2^6 b. $C(6, 4)$

25. 12

26. There are 5 flavors of Jolly Ranchers: Grape, Apple, Watermelon, Cherry and Blue Raspberry

a. $C(14, 4) = C(14, 10)$ b. $5/5^4 = 1/5^3$ c. $(5 \cdot 4 \cdot 3 \cdot 2)/5^4 = 24/125$ d. $(1 \cdot 5 \cdot 1 \cdot 1)/5^4 = 1/5^3$

e. $(1 \cdot 5 \cdot 5 \cdot 5 + 5 \cdot 5 \cdot 1 \cdot 1 - 1 \cdot 5 \cdot 1 \cdot 1)/5^4 = 29/125$ f. 11

27. a. $C(15, 5)$ b. $C(7, 3) \cdot C(8, 2)$ c. $C(7, 3) \cdot C(8, 2) / C(15, 5)$ d. $5!$

e. $(1 \cdot 4 \cdot 3 \cdot 2 \cdot 1) / (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 1/5$ f. $1/5!$

28. a. $\frac{13!}{2! \cdot 4! \cdot 2!} = 64,864,800$ b. $\frac{40,320}{64,864,800} = \frac{8! \cdot 2! \cdot 4! \cdot 2!}{13!}$