

Discrete math practice problems:

1. Induction practice. There will be one induction proof on the test, and it will be a series (sum of a sequence) like one of these two sample problems:

a. $7 + 12 + \dots + (5n + 2) = \frac{n(5n + 9)}{2}$ b. $1 + 7 + \dots + (2n^2 - 1) = \frac{n(2n - 1)(n + 2)}{3}$

c. In addition, I may ask you to show that you understand a statement about a series, so if I tell you that we will prove the statement: $S(n): 7 + 12 + \dots + (5n + 2) = \frac{n(5n + 9)}{2}$, I may also ask you to verify a specific case. For example: state and verify $S(3)$.

2. From the set: {a, b, c, d, e, f}

- a. How many subsets does the set have?
- b. How many 4-element subsets does the set have?

3. There are 90 widgets that need to be assembled by 8 workers. What is the smallest number that the most efficient worker (the one who assembles the most widgets) could assemble?

4. There are 5 flavors of Jolly Ranchers: Grape, Apple, Watermelon, Cherry and Blue Raspberry

- a. If I grab 10 Jolly Ranchers at random out of a bowl, how many different combinations could I get?
- b. If I randomly choose 4 Jolly Ranchers, what is the probability that they are all the same flavor?
- c. If I randomly choose 4 Jolly Ranchers, what is the probability that they are 4 different flavors?
- d. If I give Jolly ranchers to 4 people (randomly) what is the probability that the first person gets apple, **and** the last 2 people get grape?
- d'. If I give Jolly ranchers to 4 people (randomly) what is the probability that the first person gets apple, **or** the last 2 people get grape?
- e. If I grab some Jolly Ranchers without looking, how many do I need to get to be sure I will have at least 3 of the same flavor?

5. I have a stack of 15 different Pokemon cards. 7 are water type and 8 are fire type. Assume each has a different number of HP (so they can all be distinguished)

- a. In how many ways can I choose 5 cards?
- b. In how many ways can I choose 3 water type and 2 fire type cards?
- c. If I choose 5 cards at random, what is the probability that 3 are water type and 2 are fire type?
- d. If I put down 5 cards, one at a time, how many orders are there?
- e. If I put down 5 cards in a row, what is the probability that the first card has the highest HP?
- f. If I put down 5 cards in a row, what is the probability that they are in order of decreasing HP?

6. a. How many distinguishable rearrangements are there of the word Massachusetts?

b. What is the probability in a rearrangement of Massachusetts that both of the a's will be together, all of the s's will be together and both of the t's will be together?

7. Use the factorial formula to prove that $C(n, 3) + C(n, 4) = C(n + 1, 4)$ for $n > 4$.