Prove by induction that:

$S(n):\ 2+10+24+\ldots+(3n^2-n)=n^2(n+1)$ for $n\geq 1$

Proof:

$$2=2$$

$$1^2(1+1)=1\cdot 2=2$$

Check LHS = RHS for S(1)

so $S(1):\ 2=1^2(1+1)$ is true

Assume: $S(k):\ \boxed{2+10+\ldots+(3k^2-k)}=k^2(k+1)$ is true (induction hypothesis)

$LHS:\ 2+10+\ldots+(3(k+1)^2-(k+1))=$

$\boxed{2+10+\ldots+(3k^2-k)}+(3(k+1)^2-(k+1))=$

LHS of S(k)

$k^2(k+1)\ +\ (3(k+1)^2-(k+1))=$

$(k+1)(k^2+3(k+1)-1)=(k+1)(k^2+3k+2)=$

$k^3+3k^2+2k+k^2+3k+2=k^3+4k^2+5k+2$

LHS of S(k+1) & simplify

$RHS:(k+1)^2((k+1)+1)=(k+1)^2(k+2)=$

$(k+1)(k+1)(k+2)=(k+1)(k^2+3k+2)=$

$k^3+3k^2+2k+k^2+3k+2=k^3+4k^2+5k+2$

RHS of S(k+1) make same as by simplifying/algebra

so: $S(k+1):\ 2+10+\ldots+(3(k+1)^2-(k+1))=(k+1)^2((k+1)+1)$

Therefore, by induction, $S(n):\ 2+10+24+\ldots+(3n^2-n)=n^2(n+1)$ is true for all $n\geq 1$

$S(k):\ 2+10+\ldots+(3k^2-k)=k^2(k+1)$

$S(k+1):\ 2+10+\ldots+(3(k+1)^2-(k+1))=(k+1)^2((k+1)+1)$