Math 236 Test 1 review

1. List an element and a subset of each of these sets:

a. $C^1(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is differentiable}\}$

b. $C^1(\mathbb{R}) \times \mathbb{Z}$

c. $2^{\mathbb{Z}}$

d. $E = \{\{(x,y) \in \mathbb{R}^2 \mid (ax)^2 + (by)^2 = 1\} \mid a,b \in \mathbb{R}\}$

2. For each of these statements, tell whether it is true or false. If it is false, tell why.

a. $\{0.5, \sqrt{2}\} \in 2^{\mathbb{R}}$ b. $\{0.5, \sqrt{2}\} \in \mathbb{R} \times \mathbb{R}$ c. $\{0.5, \sqrt{2}\} \subseteq 2^{\mathbb{R}}$ d. $\{0.5, \sqrt{2}\} \in 2^{\mathbb{Q}}$

3. Tell a domain and a codomain that would make sense for each function:

a. $f(x,y,z) = (x, y+z)$ b. . $g(a+bi) = b$ c. $F(f(x),a) = f'(a)$

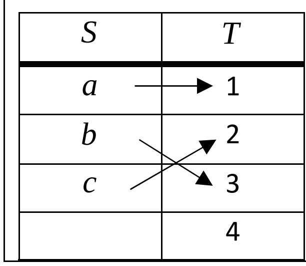
4. For the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that $f(x,y) = (y, 2x)$

Find the images: $f(3,1)$ and $f^{\to}(\{(x,y) \in \mathbb{R}^2 \mid y = x^2\})$

Find the pre-images: $f^{\leftarrow}(5,2)$ and $f^{\leftarrow}\{(a,b) \in \mathbb{R}^2 \mid b = 4a+1\}$

5. What has to be true about functions f and g in order for $f \circ g$ to make sense?

6. For each function, tell whether it is one-to-one and whether it is onto, and explain why or why not.

<table>
<tr>
<td>a. $f : S \to T$ such that

</td>
<td>b. $L = \{f(x) = ax + b \mid a,b \in \mathbb{R}\}$ is the set of degree 1 polynomials
$H = \{g(x) = c \mid c \in \mathbb{R}\}$ is the set of degree 0 polynomials
$F : L \to H$ such that $F(f(x)) = f'(x)$</td>
<td>c. $g : \mathbb{Z} \to \mathbb{Z}_8$ such that $g(n) = [n]_8$</td>
<td>d. $h : \mathbb{R}^2 \to \mathbb{R}^2$ such that
$h(x,y) = (2+x, 2^y)$</td>
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7. What properties need to be satisfied by a function f for it to be invertible?

8. Find the inverse function for each of these functions:

a. $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that $f(x,y) = (2y, x+y)$	b. $g : L \to \mathbb{R}^2$ for $L = \{ax+b \mid a,b \in \mathbb{R}\}$ Such that $g(ax+b) = (b, a+b)$	c. $h : \mathbb{R} \to (0,\infty)$ such that $h(x) = 2^x$

9. Sketch the Venn diagram to show the following set relationships.

a. $(A \cup B) \cap C$ b. $A \cup \overline{B}$ c. $\overline{A \cap B}$ d. $A \cap (\overline{B \cup C})$

10. Make truth tables for each of these, and tell which, if any, are equivalent:

a. $p \to (q \wedge r)$ b. $(\sim q) \wedge (\sim r) \to \sim p$ c. $\sim (p \wedge (\sim (q \wedge r))$ d. $((\sim q) \vee (\sim r)) \wedge p$

11. Write the negation of each statement:

a. Each point in the set lies above the x-axis

b. No function in the set is a polynomial

c. All of the dice rolled the same number

d. The numbers in the set are both positive and even

e. The numbers in the set are positive or even.

12. Write the contrapositive of each statement:

a. If a monster is a grue, then it is not happy.

b. If a polygon is starlike, then it is both compact and simplicial.

c. If a number is constructible or solvable then it is algebraic.

13-14: for each statement, circle the equivalent statements:

13. If it is a square, then it is a rectangle. (2 correct answers)

- a. All squares are rectangles
- b. All rectangles are squares
- c. Some squares are rectangles
- d. All non-rectangles are non-squares
- e. All non-squares are non-rectangles

14. Every convergent sequence is Cauchy and bounded (2 maybe 3 correct answers)

- a. If a sequence is Cauchy then it is convergent and bounded
- b. If a sequence is bounded, then it is Cauchy and convergent
- c. If a sequence is convergent then it is Cauchy and bounded
- d. If a sequence is not Cauchy or not bounded then it is not convergent
- e. If a sequence is both not Cauchy and not bounded then it is not convergent
- f. If a sequence is not both Cauchy and bounded then it is not convergent

15-18: Prove each statement

15. If k divides n and $a \equiv b \pmod{n}$ then $a \equiv b \pmod{k}$

16. The sum of an even integer and an odd integer is odd.

17. If $a = bc + r$ and d divides both a and b then d divides r

18. If $x + y > 50$ then $x > 10$ or $y > 40$

Definitions and formulas:

$x \equiv y \pmod{n}$ means that $x = y + kn$ (where k is some integer)

An integer n is even means $n = 2k$ for some integer k

An integer n is odd means $n = 2k + 1$ for some integer k

n divides m means that n divides evenly into m, and n is a factor of m. Algebraically it means that $m = nk$ for some integer k