Given: $S_n = 3S_{n-1} - 1 \qquad S_0 = 2$

Prove: $S_n = \left(\frac{3}{2}\right)3^n + \frac{1}{2}$

<u>Proof</u>: $S_0 = \left(\frac{3}{2}\right)3^0 + \frac{1}{2} = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$ ✓

(If) Assume: $S_k = \underbrace{\left(\frac{3}{2}\right)\cdot 3^k + \frac{1}{2}}$

Then: $S_{k+1} = 3S_k - 1$

$$= 3\left(\left(\frac{3}{2}\right)3^k + \frac{1}{2}\right) - 1$$

$$= 3^1\cdot\left(\frac{3}{2}\right)\cdot 3^k + \frac{3}{2} - 1$$

$$= 3^{k+1}\left(\frac{3}{2}\right) + \frac{1}{2} = \left(\frac{3}{2}\right)3^{k+1} + \frac{1}{2}$$

Therefore: $S_n = \left(\frac{3}{2}\right)3^n + \frac{1}{2}$ for $n \geq 0$

Homework: Prove by induction (turn in Mon)

(1) Given: $S_n = 3S_{n-1} + 4$; $S_0 = 5$

Prove: $S_n = 7\cdot 3^n - 2$

(2) Given: $S_n = 3S_{n-1} - 5$; $S_0 = 4$

Prove: $S_n = \left(\frac{3}{2}\right)3^n + \frac{5}{2}$

or $S_n = \frac{3(3^n) + 5}{2}$