

Math 236 – Discrete Mathematics  
Section A.1: Statements and Connectives

A statement is either true or false.

Examples:

- 1) Six is an integer.  $\top$
- 2)  $\pi$  is an integer.  $\text{F}$
- 3) All integers are real numbers.  $\top$
- 4) All real numbers are integers.  $\text{F}$
- 5) Some real numbers are integers.  $\top$

"Some" = "at least one"

Non-examples:

- 1) Do your homework.
- 2) Did you do your homework?

Ambiguous statement example:  
Pizza is good.

CONNECTIVES

Colloquial Term	Symbol	Technical Term
not	$\sim$	negation
and	$\wedge$	conjunction
or	$\vee$	disjunction
if...then	$\Rightarrow$	conditional
if and only if	$\Leftrightarrow$	biconditional

NEGATIONS

Find the negation of the following statements.

The negation of a statement is the statement itself.

- 1)  $s$ : My cat has three ears.

$\sim s$ : My cat does not have 3 ears.

$$\sim(\sim s) = s$$

- ✦ 2)  $s$ : All dogs are black.

$\sim s$ : Not all dogs are black.

$\sim s$ : Some dogs are not black.

- ✦ 3)  $s$ : Some quadrilaterals have exactly three sides.

$\sim s$ : All quadrilaterals do not have exactly 3 sides.

$\sim s$ : No quadrilaterals have exactly 3 sides.

- 4)  $p$ : My hair is brown.      $q$ : My eyes are blue.

$\sim p$ : My hair is not brown      $\sim q$ : my eyes are not blue

$\sim(p \wedge q)$ : I do not have both brown hair and blue eyes

$\sim p \vee \sim q$ : My hair is not brown or my eyes are not blue

$\sim(p \vee q)$ : I don't have either brown hair or blue eyes

$\sim p \wedge \sim q$ : My hair is not brown and my eyes are not blue

$p \rightarrow q$ : If my hair is brown then my eyes are blue

$p \Leftrightarrow q$

Logically equivalent  
De Morgan's Laws

np 19)

My hair is not brown or my eyes are not blue . ☺

My hair is not brown and my eyes are not blue ☹

I do not have both brown hair and blue eyes ☺

$P \vee Q$ : my hair is brown or my eyes are blue

$\sim(P \vee Q)$ : ~~my hair is not brown or my eyes are not blue~~

I do not have brown hair or blue eyes  $\leftarrow$

\*  $\rightarrow$  My hair is not brown and my eyes are not blue

\* I don't have either brown hair or blue eyes  $\leftarrow$

almost the same,  
"either" is more clear

$$(\sim (p \vee q)) \rightarrow p$$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim(p \vee q) \rightarrow p$
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

