

Prove there is no  
H-cycle

• Vert. 1, 5, 3 have to have  
2 edges each

SO

Have to use a, b

c, d

e, f

that makes 4 adjacent  
to 3 edges.

So it can't be a  
Ham. cycle

Other proof technique

✓ 4 is adjacent to 3  
edges.

only allowed to use 2  
(-1)<sup>†</sup>

✓ 2 is adjacent to 3  
edges, only allowed  
2 (-1)<sup>†</sup>

4 = 6 - 2<sup>†</sup> edges I am allowed  
to use

I have 5 vertices, only 4 allowed edges

I can't make a Ham. cycle.

\* 6 = 6 total edges in the graph

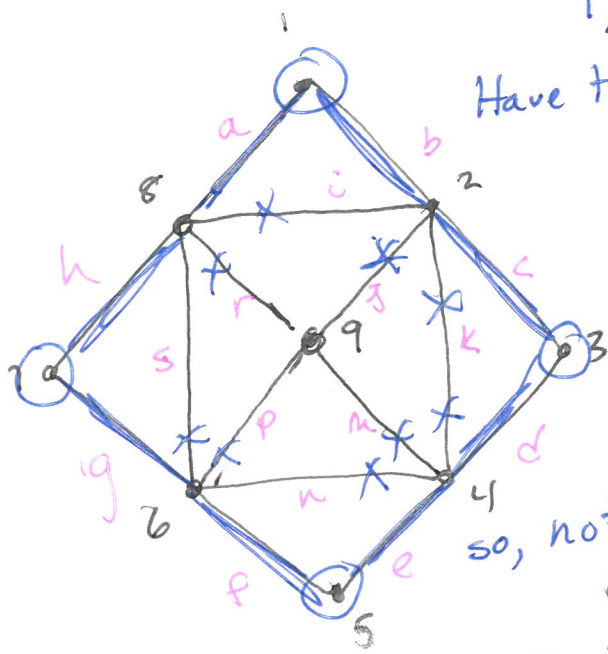
† -2% 2 cannot be used

(one of a, c, e, one of b, d, f)

A Hamiltonian cycle must have

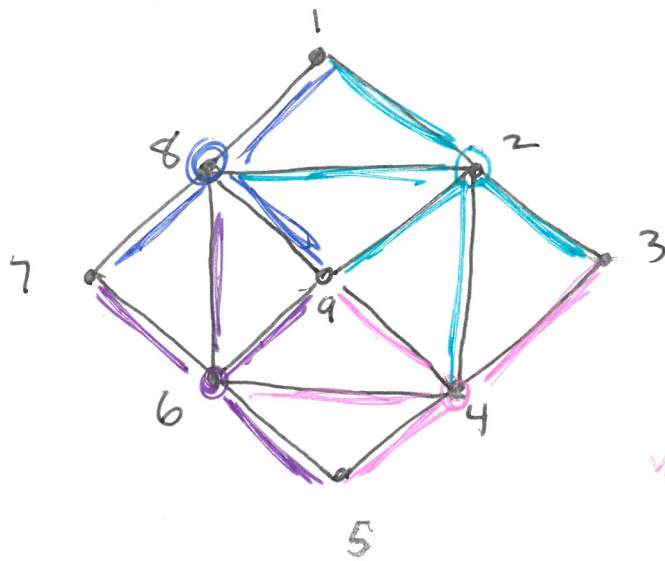
• each vertex adjacent to 2 edges

• Number of vertices = number of edges in the cycle



1, 3, 5, 7 have deg. 2.  
 Have to use edges a-h

if I use a-h  
 I can't use any  
 other edges adjacent  
 to 2, 4, 6, 8  
 so, not allowed any other  
 edges.  
 can't get to v. 9  
 so no Ham. cycle



9 vertices

16 edges

v. 2 is adjacent to 5 edges.  
only use 2

-3

v. 4 is adj to 4 new edges. only use 2

-2

v. 8 is adj to 3 new edges. only use 2

-1

v. 6 is adj to 4 new edges

only use 2

-2

Edges we're allowed to use:

$$16 - 3 - 2 - 2 - 1 = 8 \text{ edges} < 9 \text{ vert.}$$

Not enough edges to make a Ham Cycle