A. The product of an even integer and an odd integer is even.

Given: An even integer, n, and an odd integer m,

Then: The product, $n \cdot m$, is even

proof:

n is even, so $n = 2k$ $(k \in \mathbb{Z})$

m is odd, so $m = 2h+1$ $(h \in \mathbb{Z})$

$$n \cdot m = 2k(2h+1) = \cancel{2}\, 4kh + 2k = 2(2kh+k)$$

$$n \cdot m = 2(k(2h+1))$$

so $n \cdot m$ is even $\square$

Given

B. If a divides b and a divides c then ~~a divides bk+cj~~

a divides bk+cj to prove

Proof: a divides b, so $b = ar$

a divides c, so $c = ah$

$bk + cj = (ar)k + (ah)j = ark + ahj$

$= a(rk + hj)$

$bk + cj = a(rk + hj)$

a divides bk+cj

c. If $a \equiv c \pmod{n}$ and $b \equiv d \pmod{n}$

then $a+b \equiv c+d \pmod{n}$

$a \equiv c \pmod{n}$ so $a = c + nk$

$b \equiv d \pmod{n}$ so $b = d + nj$,

$$a+b = c+nk + d+nj$$
$$= c + d + nk + nj$$
$$a+b = c+d+n(k+j)$$

so $a+b \equiv c+d \pmod{n}$ $\square$