To prove:

$$C(n,r) + C(n,r+1) = C(n+1,r+1)$$

RHS: $C(n+1,r+1) = \dfrac{(n+1)!}{((n+1)-(r+1))!\,(r+1)!} = \dfrac{(n+1)!}{(n-r)!\,(r+1)!}$ ☆

LHS: $C(n,r) + C(n,r+1) = \dfrac{n!}{(n-r)!\,r!} + \dfrac{n!}{(n-(r+1))!\,(r+1)!}$

$$= \frac{(r+1)}{(r+1)}\frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r-1)!\,\underbrace{(r+1)!}_{=(r+1)(r!)}}$$

$$= \frac{n!(r+1)}{(n-r)!\,(r+1)!} + \frac{n!}{(n-r-1)!\,(r+1)!}\,\frac{(n-r)}{(n-r)}$$

$$= \frac{n!\,(r+1)}{(n-r)!\,(r+1)!} + \frac{n!\,(n-r)}{(n-r)!\,(r+1)!}$$

Factor out

$$= \frac{n!\,(r+1) + n!\,(n-r)}{(n-r)!\,(r+1)!}$$

$$= \frac{n!\,\big((r+1)+(n-r)\big)}{(n-r)!\,(r+1)!}$$

1 more

$$= \frac{n!\,(n+1)}{(n-r)!\,(r+1)!} = \frac{(n+1)!}{(n-r)!\,(r+1)!}$$ ☆

bigger $(r+1)r! = (r+1)!$ ☺

smaller $(n-r-1)(n-r)! \neq (n-r-1)!$

$(n-r)! = (n-r)(n-r-1)!$ ☺

$6 \cdot 5! = 6!$

$6(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$

$4 \cdot 5! \neq 4!$

$5! = 5(4!)$ ☺

So $C(n,r) + C(n,r+1) = C(n+1, r+1)$

pg 77 Example 2.49 ← Read

Prove: $5+8+11+\dots+(3n+2) = \frac{1}{2}(3n^2+7n)$

HW pg 406 (8.1)

#26, 29, 30 use the $C(n,r)$ formula

#30 hint

$$C(2n,2) = \frac{(2n)!}{(2n-2)!\,2!}$$

$$= \frac{(2n)(2n-1)}{2!}$$