Discrete Math Final Exam Study List, Spring 2018: Answers to some problems

2a. Not isomorphic: the top graph has two 3-cycles (ABF and CDE), and the bottom graph has no 3-cycles

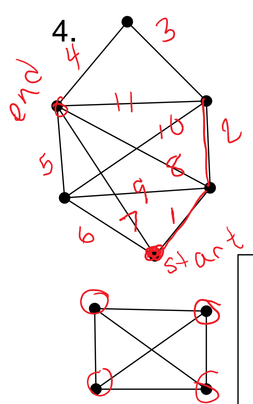
2a. Isomorphic: M→W N→V O→T P→U Q→S R→X

3. Know how many edges K_n has. $n(n-1)/2$

4. Know how many edges a tree with *n* vertices has. $n-1$

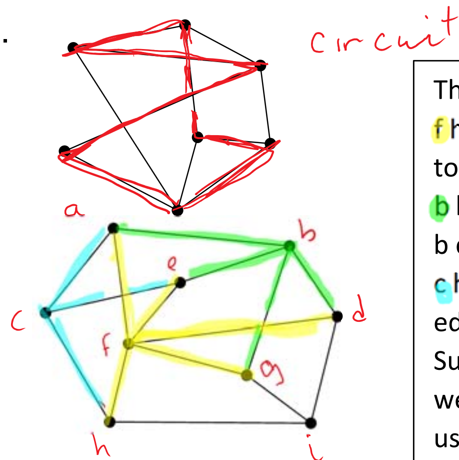
I mis-labelled. The "4" graphs were intended for Euler paths, and the 5 graphs were intended for Hamiltonian circuits

4.



More than 2 vertices have an odd degree, so no Euler path/circuit

5.



There are 9 vertices and 14 edges.
f has degree 5, so 3 edges adjacent to can't be used (14-3=11 edges)
b has degree 4, so 2 edges next to b can't be used (11-2=9)
c has degree 3 (no overlap in the edges), so 1 edge can't be used
Subtracting from the total edges, we find 8 (or fewer) edges can be used, which is not enough for a Hamiltonian circuit

18. Every convergent sequence is Cauchy and bounded
    a. If a sequence is Cauchy then it is convergent and bounded
    b. If a sequence is convergent then it is Cauchy and bounded
    c. If a sequence is not convergent then it is not Cauchy or not bounded.
    d. If a sequence is not Cauchy or not bounded then it is not convergent
    e. If a sequence is both not Cauchy and not bounded then it is not convergent
    f. If a sequence is not both Cauchy and bounded then it is not convergent

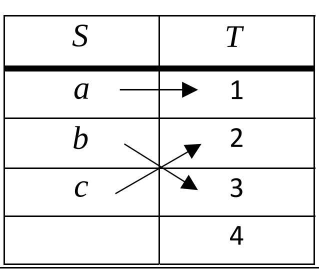
21. For the function $f:\mathbb{R}^2 \to \mathbb{R}^2$ such that $f(x,y)=(x+y,x^2)$

Find the image: $f(3,4)=(3+4,3^2)=(7,9)$ and the pre-image: $f^{\leftarrow}(5,4)$

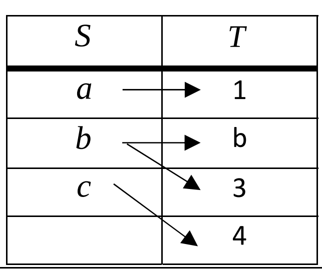
To find the pre-image, set: $f(x,y)=(x+y,x^2)=(5,4)$ so $x+y=5$ and $x^2=4$ Because $x^2=4$, $x=\pm 2$
If $x=2$ then $2+y=5$ so $y=3$. If $x=-2$ then $-2+y=5$ so $y=7$. The pre-image is $\{(2,3),(-2,7)\}$

22. For each of the following, tell if it is a. a function, b. one-to-one, c. onto

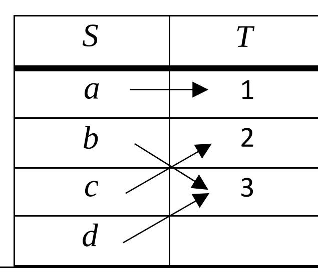
A. $f:S\to T$ such that



B $f:S\to T$ such that



C. $f:S\to T$ such that



A. is a 1-to-1 function, and is not onto; B. is not a function (it is a 1-to-1, onto relation) C. is an onto function that is not 1-to-1.

23. Compute: a. $4\times 5 \equiv 6 \pmod 7$ b. $2-8\equiv 4 \pmod{10}$ c. $6\times 4+7\equiv 4 \pmod 9$